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ABSTRACT

This document consists of papers presented at the Second Mathematics Education Conference in May 1982 at Lae, Papua New Guinea. Included are the following titles: (1) The Use of Academic Subjects in a Service Capacity; (2) Some Factors Affecting Mathematics Learning in a National High School; (3) An Extraneous Role for the VTR in Educational Research; (4) A Problem Solving Project - How to Do It; (5) In Algebra We Do Operations on Letters; (6) The Case for Changes in Approaches to Mathematics Curricula: Some Suggestions for Research and Teaching; (7) A Preliminary Study of the Mathematical Generalization Process in Late-Secondary Papua New Guinean Students; (8) The Rotation and Development of Surfaces - A Study of a Piagetian Spatial Task with PNG Secondary School Pupils; (9) Attitudes to Success and Failure in Mathematics; (10) Community School Mathematics: The International Connection; (11) The Use of Algorithms in Year 7; (12) Computer-Assisted Instruction at Unitech; (13) The Development of Time Concepts and the Use of Comparatives among Papua New Guineans; (14) Regional Cooperation: PEACESAT, PRIME and the SPMLC; and (15) Development and Implementation of Math Textbooks in PNG. The document concludes with a program outline of the conference. (MP)

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RESEARCH IN MATHEMATICS EDUCATION

IN

PAPUA NEW GUINEA:

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THE USE OF ACADEMIC SUBJECTS IN A SERVICE CAPACITY - SOME PROBLEMS OF STATUS

PETER E. CLARKSON

WARRAGUL TECHNICAL SCHOOL

Problems are created for educators because of the different status afforded to different types of knowledge and to the persons who engage in these forms of knowledge.

Creators of knowledge, often called academics, and the knowledge that results from their work, are generally given high status while lower status is allocated to the users of this knowledge in applied or practical form.

This status difference also appears in the field of education where liberal education is generally preferred to vocational education.

Yet in any industrial society or in a society moving toward industrialisation it is recognised that pure knowledge must be applied to specific purposes. The design and implementation of special educational programs is part of the process of achieving this aim.

Involvement in such programs creates problems for educators who have been trained and who work as academics. Having to submit to the dictates of what may be seen as a lower status area of knowledge could result in the production of curricula that are inappropriate for the purposes for which they were intended.

A possible solution to the problem may be found through the involvement of persons who have a special blend of academic knowledge and vocational training.

THE USE OF ACADEMIC SUBJECTS IN A SERVICE CAPACITY - SOME PROBLEMS OF STATUS

Introduction

This paper will consider the existance and characteristics of forms of knowledge in Western society that are given both high and low status by the society. The relationship between knowledge and education will be explored as will the influence of industrial society on this relationship. A reminder will be given of the special elements educators must consider in the structuring of educational programs for use in a democratic society.

The requirements and characteristics of technical and vocational education will be discussed plus the special problems that are created when academics are involved in curriculum design for this form of education.

Finally the paper will consider the involvement of various types of persons in curriculum design and will make a judgement as to which of these would be most likely to lead to the design of an effective vocational curriculum.

Knowledge

Knowledge can be defined as 'The forms of meaning through which human life and the world are described, explained, interpreted and evaluated and through which human experience is expressed'. [1] Each society developes its own collection of meanings or 'stock of knowledge' [2] to use Musgraves term. This knowledge is transmitted from one generation to the next through the process of education. Each generation may develop additional forms of meaning or acquire them from other societies. Each generation may discard some forms of meaning as their experience of the world differs from preceeding generations. These processes thus produce changes in the stock of knowledge possessed by each society.

In simple societies the stock of knowledge was common to all members. In large and complex societies the stock of knowledge is shared between different groups.

In western society knowledge has been divided up in many ways. One particular division, to be explored in this paper, is the division of knowledge in to high status knowledge and low status knowledge. [3]

This division can be traced back to the foundations of Western knowledge in ancient Greece. In this society, knowledge resulting from cognitive or intellectual activity was preferred to knowledge related to physical work.

In this way knowledge related to thinking received high status while knowledge related to doing received low status. The high status knowledge was non-instrumental in that it was not required, for its justification, to be directed toward a specific useful end. This form of knowledge was considered to have intrinsic value. Instrumental knowledge related to work involved the application of high status knowledge to specific objectives. Instrumental activities used but did not create knowledge.

In ancient Greece the high status knowledge was possessed by a relatively small section of the population who were also the rulers of that society. The majority of the population, many of whom were slaves, were the possessors of the lower status knowledge. Thus the division of the stock of knowledge in this way was also related to social power and status.

These status differences were maintained through the subsequent stages development of Western society. Small groups or elites in Western society, sometimes the clergy, sometimes the aristocracy, sometimes the universities, were the possessors of high status knowledge while the majority of the population possessed the low status knowledge.

To this day countries like Australia and Papua New Guinea are still influenced by these traditions which were transmitted through the original colonial contact with England.[4]

Knowledge, Education and the Development of the Industrial Society

In the broad sense education is the handing on of the stock of knowledge in a society, from one generation to the next including the means for creating new knowledge to be added to the stock. Simply by living in a society, new members, children, will acquire some knowledge through contact with the older members but generally in any society the adult members will need to make a deliberate effort to ensure that the new members will acquire distinctly human ways of thinking, feeling, imagining, communicating, acting and so on. In modern societies this process is so complex that it is entrusted to specially qualified persons, teachers, who carry out the process in organisations called schools.[5]

The portions of the stock of knowledge to be handed on to different members of society resulted in different forms of education that were delivered in different types of schools.

Education related to intellectual or non-instrumental knowledge was termed liberal education while education related to instrumental knowledge was termed vocational education. Generally in Western society and in line with the previous discussion on knowledge, liberal education was afforded high status while vocational education was given low status. [6]

In the initial stages of Western society there was little need for the masses to possess any high status knowledge at all. Thus only the elite were educated to read, write, understand mathematics and so on but with the rise of the industrial state from the 18th Century onwards this situation altered. The need for the masses to be employed in various technological process required that this group of society be given access to some high status knowledge in the form of basic literacy and numeracy. Gradually this portion of high status knowledge was delivered through a vocational education process but as such it lacked the totality of true high status knowledge in that it was to be used for a specific end and was not to be used in the creation of further knowledge. Thus with the rise of industralisation the masses gained limited access to high status knowledge which enabled them to live and work in the new industrial society.

Education Knowledge and Democracy

At this point it may be useful to digress slightly to look at some matters related to education, knowledge and democracy. Young [7] has pointed out that there are important considerations of power and social control involved in the selection and organisation of knowledge in any educational program or curriculum. Considerations of power and control are also central to the ideals of democracy. Thus in the planning of any curriculum to be used in a democracy care must be taken to ensure that such programs do not contradict the democratic aims of the nation in which the programs are to be used. The following discussion of this topic must be brief, in a paper of this type, but I believe the point should be noted.

Democracy refers to ideas of Freedom, Equality and Community and offers a way of structuring and organising human society, so that the power of that society is shared equally among the individual members. Education has a special relationship to democracy because if all the members of a society are to share power and have an influence on what that society does then all members must be equipped with the appropriate knowledge. A society, which tolerates a situation in which distribution of knowledge is not equal and where members who possess a certain type of knowledge are afforded a higher status than other members who possess a different type of knowledge, could not strictly be classed as democratic.

Societies who call themselves democratic have recognised this contradiction and have made attempts to rectify the situation. Two such attempts involving education will be mentioned.

Comprehensive education represents one of the attempted solutions.[8] In a comprehensive education program the curriculum includes both liberal and vocational education which is made available to all students ideally up to year 12. Musgrave sees this approach as an attempt 'to redefine secondary education as an entity in itself that gives all children access to as wide a range of knowledge as possible without any historically determined connotation of different status.'[9] Beyond the end of secondary schooling it is recognised that the needs of society will demand that persons specialise in activities requiring a predominance of one of the forms of knowledge. What it is hoped has been achieved at the end of schooling is that all students will appreciate and value both forms of knowledge and may even have integrated the two forms for themselves. Thus it is hoped that all students will have acquired the means to gain access to both forms of knowledge throughout their lives no matter in what form their vocation causes them to specialise.

A second approach to the problem of high and low status knowledge is to try and eliminate the status difference while continuing to recognise the two forms. This has been tried in the U.S.S.R. where positive discrimination has been directed in favour of instrumental knowledge in an effort to raise it to be equal in status with academic knowledge.[10]

Both these attempts are progressing slowly, and the resistance to such a fundamental change is great. Nevertheless the existance of a status hierarchy in knowledge will continue to be a special problem in democracies particularly in those nations who are, or who are developing as, industrial nations. The important point is that educationalists should be aware of the problem and should aim to overcome it and not make the problem worse through any programs they design.

Vocational and Technical Education

Democratic nations who choose to base their societies on an industrial and technological foundation create for themselves a dilema. The industrial and technological processes demand that the majority of the society possess instrumental knowledge, while the minority possess academic knowledge. Because of the historical status difference between these two forms of knowledge the possession of such forms creates status differences which can be seen to contradict the democratic ideal.

But democratic nations must also face the fact that if they are to remain or develop as industrial nations they must organise their education systems to ensure that instrumental knowledge is passed on to a high proportion of the population. This is what may be termed a pragmatic view of the situation and if this view is accepted the design of vocational education programs must be handled carefully knowing that within them may exist the potential for long term harm to democracy. Thus perhaps something like the comprehensive education approach should preceed any entry to vocational education.

The major features of instrumental knowledge are that it is used to achieve a specific end and that it uses rather than creates new knowledge. The knowledge that it uses includes aspects of academic knowledge as well as manual and other skills. Thus curricula for technical and vocational education tend to embody specific aims or behavioural objectives aimed at producing specific behaviour or skills to be used in special circumstances, e.g. Plumbers, Doctors, Technicians, Engineers.

As mentioned in relation to the design of vocational and technical curricula, various components of academic knowledge are required. Frequently and quite logically the persons asked to advise and contribute to this aspect of curriculum are persons who possess the required form of academic knowledge, i.e. academics. This arrangement creates a special problem.

Academics involved in the preparation of Voctaional and Technical Curricula

Nations existing or expecting to exist on the basis of technology accept the need to see that certain members of their society acquire instrumental or practical knowledge. This is done with the aid of a curriculum designed to pass on the required knowledge.

The design and content of this type of curriculum is characterised by certain features.

1. It usually has reasonably precise objectives
2. It is aimed at the use rather than the creation of knowledge
3. It uses academic knowledge in an incomplete sense.
4. It uses academic knowledge in an ancillary capacity.

Each of the features could create problems for any academic asked to participate in the design of such curricula. The situation in relation to each characteristic will be discussed briefly.

The highly objective and structured format of many vocational programs[11] is a difficult format for academics to relate to in that it is in many ways opposite to an academic background where the stress is on an open and non-instrumental approach to learning and knowledge. Yet if an academic is to be involved it must be accepted that the aim is to teach a separate skill or technique and that instincts in the opposite direction must be contained.

Similarly the use of knowledge rather than the creation of knowledge would have to be dealt with in a similar manner, although this step may not be so difficult as most academics use as well as create knowledge.

The use of academic knowledge in an incomplete form is a difficult feature for an academic to come to terms with. It leads to charges that curriculum components produced by an academic for a vocational curriculum are 'too academic',

'the material is over the students head' and so on. Academics appreciate the wholeness of their disciplines and to be asked to contribute only 'bits' of that knowledge is difficult. The academic must once again realise that 'bits' are all that is wanted. The central aim of the vocational curriculum is instrumental knowledge not academic knowledge. The temptation to increase the academic content to 'round off the topic' must in most cases be resisted by the academic.

The use of academic knowledge in an ancillary or service capacity is the final feature the academic must come to grips with. This is probably the most difficult feature from an academics point of view. Not only is the academic required to only contribute 'bits' of his knowledge he must also accept that his knowledge is not the focus of the exercise. The knowledge he contributes is only a means to an instrumental end. A difficult position for a person with a stock of high status knowledge, worthwhile in itself, who sees this knowledge in effect reduced in status because academic knowledge is now being used at the direction of the requirements of what in general is lower status knowledge. Again the academic must adjust because in most cases what is happening is exaclty what is required by a curriculum whose major concern is instrumental knowledge.

A further factor, that may not actually form a formal part of any curriculum but which is an informal part of most vocational processes, is the accepted use of common sense. Common sense may be seen as a crude type of inductive or deductive thinking where limited evidence is gathered and on the basis of this evidence a conclusion is reached. For an academic schooled in the ways of assembling evidence, logic and so on the common sense approach often offends. But again this is an approach that is useful in terms of instrumental knowledge and must be accepted accordingly by academics involved.

There are probably other areas of concern for academics involved in the presentation of curricula related to instrumental knowledge, but the ones discussed should serve to highlight the sort of readjustment academics will need to make if they are involved in these projects. If adjustments are not made by academics and they attempt to impose their academic values on a vocational curriculum the result is generally disastrous for the aims and conduct of such programs. What is being suggested is that academics in these circumstances must be prepared to adjust their values to suit the aims of the particular vocational program.

Involvement in Vocational Curriculum Planning - Some possible Arrangements

Previous sections of the paper have discussed some of the problems associated with the preparation of vocational curricula where academics are involved and where academic knowledge is to be part of the vocational curricula.

This section will consider some arrangements of persons [12] each bringing their particular knowledge to the process that may be involved in the planning and design of vocational curricula. A brief analysis of these arrangements will be made, in the light of previous discussions, with a view to suggesting what the most efficient arrangement might be.

Some possible arrangements are -

1. Specialists design the program
2. Specialists are given access to limited academic knowledge through in-service programs and then designs the program
3. Academics and Specialists work together to design the program
4. Academics design the program
5. Academics are given access to limited instrumental knowledge through in-service programs and then designs the program
6. Specialists, who are also academics, design the program
7. Academics, who are also specialists, design the program

Briefly analysing each arrangement the following conclusions are reached.

Arrangements 1 & 4 could be discarded as there is no guarantee both types of knowledge would be adequately represented.

Arrangements 2 & 5 are possibilities provided each can and will accept the form of knowledge in which they are deficient.

Arrangement 3 is a reasonable possibility provided both can resolve status differences in a knowledge and personal sense.

Arrangement 7 is unlikely as academics rarely become specialists.

Arrangement 6. This I would suggest is the most desirable solution. There are people who have first trained as specialists and then complete training as academics. Thus there is an integration of both types of knowledge within the one person. This arrangement would thus overcome most of the trouble areas as such people would have the capacity to evaluate a situation from both status levels.

Where the people can be obtained or how it can be arranged that they are available would be the subject of another paper but such people would be valuable in curriculum project teams and would go a long way to overcoming problems associated with the use of academic subjects in a Service capacity.

Conclusion

The problem implicit in this paper has been how to deal with academic knowledge that is used in a service capacity in vocational curricula.

It has been suggested that a major source of this problem has been the traditional status difference that exists, in Western society, between academic knowledge and instrumental or practical knowledge.

The approach used in the paper, to finding a solution, has been essentially pragmatic, in view of the short term necessity of providing instrumental knowledge to support an industrial society or a developing industrial society. In adopting this approach the two forms of knowledge and the status difference between them have been accepted. The directions toward a solution have therefore had to accommodate this fact.

The adoption of this approach while offering a possible solution to the specific problem could also be seen to generate problems, some major some minor, in other areas in the future. The situation in relation to democracy has been mentioned. It could be said the solution suggested in this paper has caused academic knowledge to be downgraded in an effort to boost the status of instrumental knowledge. This generates a negative attitude to academic knowledge which should not be the case, as this area of knowledge is a vital component of any area of education.

These and other problems can be traced to fundamental weaknesses in the dichotomous approach to knowledge used in this paper and while there is no room to pursue this question it could be profitable in conclusion to suggest another direction for the future.

The other direction involves adopting a more unified theory of knowledge. This is a harder and longer term solution, but offers more possibilities. Emery[13] has suggested that to be viable in the future, societies must adopt a view of knowledge and education that integrates theory and practice rather than dividing them. Whiteheads[14] essay on Technical Education of many years ago offers some ideas of the principles to be aimed at and indeed one small extract from this essay hints at what perhaps should be. Whitehead writes, 'The antithesis between technical and a liberal education is fallacious. There can be no adequate technical education that is not liberal, and no liberal education which is not technical: that is, no education which does not impart both technique and intellectual vision.'

To achieve this ideal will be no easy task, but to ignore the possibility is perhaps to turn one's back on a better educational option for the future.

NOTES & REFERENCES

1. Critenden, B.S. Essays on Quality in Education. (Primary Education) p.8
2. Musgrave, P.W. Knowledge Curriculum & Change. (Melbourne University) p.1
3. In this paper status will be defined as 'The preference for one thing over another.'
4. Musgrave, P.W. Knowledge Curriculum & Change. p.21
5. Critenden, B.S. Essays on Quality in Education. p.3
6. Musgrave, P.W. Knowledge Curriculum & Change. p.23
7. Young discusses this point in the essay 'Curricula as Socially Organised Knowledge' in 'Knowledge & Control.' Young, H.E.D. (ed.)(open University)
8. The term Comprehensive Education is used in a very limited sense in the context of this paper. Comprehensive Education usually makes much wider claims in relation to education and democratic society.
9. Musgrave, P.W. 'Knowledge Curriculum & Change'. p.30
10. Zajda, J.I. 'Introductory Work-Training in Soviet Schools' in UNICORN (Australian College of Education) Vol.6 No.2 1980.
11. For an example of this approach to Curriculum design see Instructural systems model as presented in 'A systems approach to training' TAFE Services Education Department Victoria.
12. In this paper the term 'specialist' will refer to a person who possesses primarily 'Instrumental Knowledge' while the term 'Academic' will refer to a person who possesses primarily academic knowledge.
13. Emery discusses this ideas in various publications, e.g.
    Educational Paradigms - F. Emery
    Education & Change - F. Emery
14. Whitehead, A.N. 'The Aims of Education & other essays'. (E. Benn Ltd.) p.74

SOME FACTORS AFFECTING THE MATHEMATICS LEARNING IN A NATIONAL HIGH SCHOOL

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Previous research at Unitech has suggested that a good predictor of mathematics is whether the students prefer to process mathematical problems analytically or visually; the analytic students performing the better. This paper reports on a longtitudinal study which focusses on this factor, among others, with national high school students as the subjects. Results suggest that these students already prefer analytic approaches to mathematics. Although this factor is a poor predictor in year 11, it seems to be more important in year 12, a trend which would fit with previous research results. However, this paper is only a preliminary report with data analysed from only one school. Such trends are only suggestive and will need to be confirmed by later analyses.

SOME FACTORS AFFECTING MATHEMATICS LEARNING IN

A

NATIONAL HIGH SCHOOL

INTRODUCTION

In a project with first year Engineering student at the PNG University of Technology, Lean and Clements (1980) found evidence to support the thesis that "there was a tendency for students who preferred to process mathematical information by verbal-logical means to outperform more visual students on both mathematical and spatial tests". This conclusion seems to be in conflict with other studies found in the literature. (Moses, 1977; Webb, 1979). However, as the authors point out their measure of mathematical performance was gained from scores on tests which were part of the students' normal course work rather than instruments composed of 'novel' problems. Even with this qualifying remark it was a result worthy of further study.

In a later study based on the data collected by Lean and Clements, Jones and Wilkens found that students who entered university after completing years eleven and twelve at National High School not surprisingly outperformed year ten entry students who completed a one year preliminary year at university before starting their degree course proper. (Wilkins, 1981). However, this difference between the two groups was not reflected in their results of the other four subjects study. In trying to explain this apparent anomaly, Jones and Wilkens found that the only significant difference between the ex-

THE INSTRUMENTS

A battery of tests were given to both year 11 and 12 students in three batches. Other scores were obtained from the school. Those considered in this study are outlined below.

Suwarsono's Mathematical Processing Instrument (VISAN).
This study's instrument was based on one developed at Monash University in 1979 by S. Suwarsono for a doctoral thesis. Suwarsono's instrument consisted of two parts; the first a series of thirty mathematical word problems and the second was a questionnaire which contained written descriptions of different methods commonly used by students who had tried to solve the problems in part 1. Questions ranged from 'very easy' to 'moderately difficult' with 'very difficult' questions being avoided. Diagrams, or suggestions that diagrams should be drawn were not included in these questions. Since a variety of methods were expected to be used by students attempting the problems, the descriptions in part 2 covered visual as well as verbal-logical methods. Suwarsono administered the questions to 200, year 7 students. He obtained a score for part 2 of the instrument using the following criteria for each problem:

+2 if the correct answer was given on part 1, and the reasoning was based on a diagram drawn by the student, or on a visual image constructed by the student.

+1 if an incorrect answer was given on part 1 and the reasoning was based on a diagram or visual image.

0 if no answer was given or the student could not decide on which method was used.

-1 if an incorrect answer was given on part 1 and the reasoning was of a verbal-logical method which did not involve the construction of a diagram or visual image.

-2 if the correct answer was given on part 1 and the reasoning was of a verbal-logical type.

were interested in research, such a bank would provide some base line data for one segment of the school population. At least one other study has made use of the data already (Hyde, 1982).

THE STUDY

The form of the study could be described as 'longtitudinal'. In 1981 grade 11 students and grade 12 major mathematics (see below) students were tested. It is proposed that in 1982 grade 12 students and first year students at least at [illegible] University of Technology, and maybe at the University of Papua New Guinea, will be tested. In 1983, first year tertiary students will again be tested. Thus two generation of students will be followed through two or three years of the critical final years of their school career, and through the transition phase to tertiary studies.

This paper reports on the first collection of data in the second half of 1981, and a preliminary analysis of results from one of the National High Schools.

THE SUBJECTS

Responses from 147 year 11 students and 128 year 12 students were analysed; 70 and 90 per cent of the total of each year. All year 11 students take five hours of mathematics per week as one of the six subject study. The course is set, with no choice between subjects. Year 12 students have an option of taking a major course, eight, 50 minute lessons per week, or a minor course, four, 50 minute lessons per week, in mathematics. Other subjects depend on the choice of students, although timetable consideration normally mean students choose a fairly straight science, arts, commercial or expressive arts stream.

year 10 and ex-year 12 groups on the variables Lean and Clements had measured was the analytic-visual processing variable. The ex-year 12 students were clearly more analytic in their preference for processing mathematical problems than the ex-year 10/preliminary year students.

It seems then that there may well be a change in the way students prefer to handle mathematical problems as they progress from year 10 through National High School. On the other hand, students who prefer to process mathematical problems by visual methods may simply drop out of the major stream of mathematics studies. This paper is the first report on a study which seeks to shed some light on this intriguing question.

AIMS OF STUDY

The starting point for this study was an interest in the analytic-visual variable used by Lean and Clements. The first question asked was whether this variable was a good predictor of mathematical performance in years 11 and 12. Secondly, what effect did students' progression through years 11 and 12 have on this variable.

Once it was decided to set up such a study, the idea of developing a data bank on students in the National High Schools (years 11 and 12), and following their performance until they started the first year of tertiary study became attractive. There has been little attempt in the past to gather global data on students in Papua New Guinea schools. Most research efforts, with one or two exceptions, have taken the form of pilot or case studies. Thus many decisions on education policy and curriculum development have been based on inadequate research data. This of course presupposes that the power brokers of education policy are interested in the results of educational research. Wilson (1981) seems rather sceptical of such an idea. It was hoped however, that for those who

Thus he obtained scores which ranged from -60 to +60. Using an extension of the Rasch model, the word problems were placed on an assumed 'analytical-visual' dimension, and this also enabled the measurement of students' preferences for visual or verbal-logical methods on the same 'analytical-visuality' scale.

Lean and Clements (1980) have given a more detailed description of Suwarsono's work. Their data supported Suwarsono's contention that the instrument did measure a person's preferred model of processing mathematical information, and that there did appear to be a distinct cognitive traint associated with mathematical processing.

For this study fourteen problems were chosen from Suwarsono's original thirty, thus giving an 'analytical-visuality' scale of -28 to +28 with a negative score indicating a preference for verbal-logical methods.

The fourteen problem test was administered to groups of students, and then they were asked to fill in the questionnaire immediately after.

Elementary Word Problems (PROPROB)

The score on the fourteen written problems which formed the basis of Suwarsono's instrument gave a measure of the students' ability for elementary mathematical written tasks.

Backward Digit Span (S3)

It was decided to include a test of short-term memory space. That chosen was the Backward Digit Span (Hiebert, J., 1979). Briefly, this test attempts to measure the maximum number of separate bits of information a person can manipulate at the same time in short term memory. The test consisted of asking a subject to repeat backwards a string of numbers which had been previously read to the subject. With ten strings per set, students progressed from repeating two numbers backwards

to a string of seven numbers, dropping out when they gave three consecutive incorrect answers. The test is administered individually.

Students were interviewed over a three day period by the author and two research assistants.

Mathematics and English Year 10 Gradings (MATH10 and ENG10)
This data was collected from the students at the beginning of the interview for the Backward Digit Span test when various individual student data was collected.

Operations Test (OPTEST)
For a measure of cognitive development, a subtest of the Operations Test (Australian Council of Educational Research, 1977) was administered by class room teachers at a convenient time to them within three weeks after the other instruments had been administered. Questions 11 to 20 and 31 to 50 were chosen. This subtest was rated for year 9-10 students in Australia according to the test's documentation. This test seemed appropriate for the subjects of this study,and could be administered in the fifty minutes available for the test.

Final Mathematics Score for the Year (MAFI81)
For the year 11 students, a common examination constructed by the Heads of the Mathematics Departments of the National High Schools is given at the end of the year and marked internally. This score, obtained from the schools, was used as the assessment of students for that year.

The year 12 students sit for an externally set and marked examination at the end of the year. This score was used as the assessment of students for that year.

Table 1 lists the seven variables used in this study.

NAME	LABEL
Visual-Analytic variable based on Suwarsono's work.	VISAN
Fourteen elementary word problems of the Suwarsono's instrument.	PROPROB
Backward digit span test for memory space.	S3
Year 10 external assessment for mathematics.	MATH10
Year 10 external assessment for english.	ENG10
Operations test measuring cognitive ability.	OPTEST
Final assessment score for year.	MAFI81

TABLE 1: Names and labels of the variables.

HYPOTHESES

It has been noted that the 'analytic-visual' variable was a good predictor of mathematical performance in the Lean and Clements (1980) study, and that Jones and Wilkins saw in this variable the explanation for the difference in performance of year 10 and 12 exit students.

The means of the variables will be compared and in particular that of VISAN to see whether the year 12 students are preferring a more analytic approach to solving mathematical problems. Secondly, to see if the VISAN is a good predictor of performance, correlations of this variable with students' final year scores will be calculated. Further a regression analysis will be carried out with the final year score as the dependent variable and others as predictor variables.

RESULTS

Comparison of Year 11 and 12

Table 2 gives the mean and standard deviations for the four comparable variables. A t-test was used to compare the means and where significant differences were found, the level of significance is shown in the table. From these results it is clear, and not unexpected, that the year 12 students were superior on the set of elementary written mathematical

VARIABLE	Year 11 $\bar{X}$	Year 11 S.D.	Year 12 $\bar{X}$	Year 12 S.D.	P
VISAN	-3.39	6.91	-2.71	7.60	N.S.
PROPROB	8.57	2.37	9.21	2.45	0.05
S3	3.17	0.94	3.54	0.78	0.01
OPTEST	24.16	4.39	26.46	3.48	0.001

TABLE 2: Means, standard deviations and level at which means are significantly different.

problems which composed Suwarsono's instrument, they had a greater short term memory space, and they apparently were operating at a higher cognitive level.

Of special interest for this study however was the 'analytic-visual' variable. The variable did not move in the expected direction from year 11 to year 12, but the difference between the years is not significant. To complicate the interpretation of this result, there is a significant difference between year 11 and 12 for the set of problems which make up the Suwarsono instrument (PROPROB). A previous study has noted this problem when comparing 'analytic-visual' scores (Clarkson, 1981). Since the 'VISAN' variable relies in part on how well students perform on the questions, a significant difference

in performance may invalidate the 'VISAN' variable. Hence the 'VISAN' variable for each student will need to be recalculated with the following scoring:

+1 if the reasoning was of a verbal logical method which did not involve the construction of a diagram or visual image.

0 if no answer was given or if the student could not decide what method was used.

-1 if the reasoning was based on a diagram drawn by the student, or on a visual image constructed by the student.

However for the rest of this report the 'VISAN' variable will only be used within a year level. Within this limit it is still a valid measure.

Correlations in Year 11

Table 3 displays the correlation coefficients for the variables in the study.

	MAFI81	VISAN	S3	PROPROB	OPTEST	MATH10	ENG10
MAFI81	1.00	-0.07	0.17	0.38	0.37	-0.52	-0.19
VISAN		1.00	-0.17	-0.20	-0.35	0.07	0.09
S3			1.00	0.15	0.28	-0.16	-0.32
PROPROB				1.00	0.29	-0.29	-0.08
OPTEST					1.00	-0.26	-0.19
MATH10						1.00	0.16
ENG10							1.00

TABLE 3: Correlation Coefficients for the Variables for Year 11 Students.

It would be expected that the final mathematics score for the students would have a positive correlation with the Operation Test and the fourteen written problems. As well, the negative correlation between the analytic-visual score and the Operation Test is understandable; a low analytic-visual score, indicating a preference for an analytic approach, links with a high score on the Operation Test which consists of short, objective questions, expecting the application of laws of mathematics for their solution. The real surprise is the coefficient of -0.52 for the score at the end of year 10 and end of year 11. In other words, a student who obtained a good score in year 10 is likely to score poorly by the end of year 11. It is tempting to drop this score out of the list of variables at this point of the study. However, this score is regarded as being important by the education community at large since it is one of the important results on which the students are selected for National High School. It would appear, however, that the abilities tested are somwhat different, although both termed mathematics.

A regression analysis was performed with the dependent variable MAFI81 (the final score for year 11). It should be noted from Table 3 that no two predictor variables have a correlation of more than, say, 0.60. Therefore problems of multicollinearity for a regression equation should not arise (Nie, et. al. 1975, p. 340). Table 4 shows the proportion of MAFI81 varience explained (multiple R^2) at each step of the analysis, the computer having been programmed to select from the remaining predictor variables the one which made the greatest contribution to the MAFI81 variance.

Only five of the six predictor variables actually entered the analysis. The significance of S3, short term memory space, was too low for further computation.

VARIABLE	MULTIPLE R^2	R^2 CHANGE	F
MATH10	0.27	0.27	24.06
OPTEST	0.33	0.06	6.36
PROPROB	0.36	0.03	6.77
ENG10	0.37	0.01	1.02
VISAN	0.38	0.01	0.94

TABLE 4: Contributions to MAFI81 Varience of Five Predictor Variables for Year 11 Students. (S3 did not enter the analysis)

Together the five variables contributed 38% of the varience in MAFI81. The MATH10 variable contributed the most (27%) with the score on the Operations Test next with 6%. If a standardized regression equation for the relationship between MAFI81 and the possible predictor variables were formed containing only those predictor variables whose estimated standardized co-efficients (Beta values) differed significantly from zero, then MATH10, OPTEST and PROPROB would all enter the equation.

Correlations in Year 12

Table 5 displays the correlation coefficients for the variables. The only reasonably strong correlation is that of the final year score with that of the fourteen written problems, which is in the expected direction.

A regression analysis was preformed with the dependent variable MAFI81 (the final score for year 12). Again it can be noted from Table 5 that no two predictor variables have a correlation of 0.60 or more. Table 6 shows the proportion of MAFI81 variance explained (multiple R^2) at each step of the analysis, the computer having been programmed to select from the remaining predictor variables the one which made the greatest contribution to the MAFI81 varience.

	MAFI81	VISAN	S3	PROPROB	OPTEST	MATH10	ENG10
MAFI81	1.00	-0.28	0.19	0.40	0.25	-0.22	-0.14
VISAN		1.00	-0.15	-0.24	0.05	0.19	0.05
S3			1.00	0.24	0.01	-0.14	-0.16
PROPROB				1.00	0.20	-0.12	-0.16
OPTEST					1.00	-0.14	-0.11
MATH10						1.00	0.31
							1.00

TABLE 5: Correlation Coefficients for Variables for Year 12 Students.

VARIABLE	MULTIPLE R^2	R^2 CHANGE	F
PROPROB	0.16	0.12	7.28
VISAN	0.20	0.04	3.51
OPTEST	0.23	0.03	3.50
MATH10	0.25	0.01	1.01
S3	0.25	0.00	0.49
ENG10	0.25	0.00	0.04

TABLE 6: Contribution to MAFI81 Varience of Six Predictor Variables for Year 12 Students.

All six variables entered the analysis. Together the six variables contributed only 25% of the variance in MAFI81. The PROPROB variables contributed 12% with the next VISAN at 4%. If a standardized regression equation for the relationship between MAFI81 and the possible predictor variables were formed containing only those predictor variables whose estimated standardized co-efficients (Beta values) differed significantly from zero, then only PROPROB would enter the equation.

DISCUSSION OF RESULTS

In comparing the variables across the year levels the only variable not to perform as expected was the one on which this study centred, the 'analytic-visual' variable. Certainly the students by half-way through their first year in National High School seem to be preferring analytic methods for processing mathematical problems. It may be that the interference from the year 12 students scoring more of the questions correct might influence this result.

There is support for the contention that the ability of students in mathematics does improve, since there is a significant increase in the mean scores on a test of cognitive ability (OPTEST), a test of short term memory space (S3), and a test consisting of written mathematical problems (PROPROB), remembering that written problems are often found difficult by these students operating in their third or forth language. Perhaps another factor, admittedly subjective in nature, which indicated that the subjects of this study were not naive mathematically speaking, was the ease with which they explained the methods they used to complete the problems.

In comparing the correlations of the predictor variables over the two years, the same pattern is evident, although the strength of correlations varies with the year. The two correlations which seemed to change were the score on the Operation Test compared with first the 'analytic-visual' variable and second with the short term memory space variable. The explanation for both these variations may lie in the fact that the Operations Test was easier for all students in year 12 possibly because of the added practice they had during year 11 with such mathematics. In other words they were more familiar with the work. Thus the advantage those year 11 students had who operated more analytically, or could manipulate more units of information in their short term memory (an assumed advantage for this test) was reduced when year 12 students were examined.

The regression analysis does not confirm the hypotheses arising from the earlier studies. The 'analytic-visual' variable was not a good predictor at year 11 and did not reach significance at year 12, although it was the second variable to enter the analysis in the latter year. This may be the beginning of a trend. If so then this variable should be a good predictor when a subset of these students are enrolled at the Papua New Guinea University of Technology.

CONCLUSION

This year the final revision of the three (year 11, year 12 Major and year 12 Minor) mathematics syllabi for the National High Schools was completed. Although there will doubtless be (and must be) continuing small revisions to these documents, their basic form is fixed for some years. Last year, at the author's suggestion, a comprehensive file was set up to to collect teaching notes, tests, exercises and revision notes for the three courses, so that in the absence of a standardized teaching manual or even adequate student texts for the courses, a record of the teaching approaches in the schools could be made and shared between them. Hopefully this file will form the basis for a future text book and/or teaching manual for the course; a necessity for the future transition to a national teaching force, and the present transitory stay of expatriates.

It is hoped that in this future writing project for National High Schools, notice is taken of studies such as this one and Hyde's (1982). What then is the message for the curriculum developers if the conjectures in this study do in fact prove to be correct?

First, although it should not have to be said, good Papua New Guinean students are capable of sophisticated mathematical thinking. Such ability is shown in this study by the majority of students clear capability of describing their methods used in solving the word problems, their increasing capacity of short term memory space, and their increasing cognitive level as measured by the Operations Test. They will be behind overseas students in some ways. For example they will not

have covered so much mathematical content in their studies, nor will they be so adapt at skills such as basic algebraic manipulations since they will not have been exposed to the vast quantity of examples which their overseas counterparts work through. However curriculum developers should not try to rapidly fill in the content which is usually covered in other countries (and may be there is already too much in the syllabi). Rather due emphasis should be given to encourage students to develop their ability to think through the mathematics. One of the real problems of a text book writer is to know when to stop adding extra, extension material. In my opinion the syllabi as now written should be regarded as the absolute maximum, not a minimum. I suspect lecturers at tertiary institutions would be happier to teach more content if their incoming students had developed better approaches to mathematics than simple memorization.

This study also raises the same questions about the use of visual methods of processing mathematical problems as an earlier study (Clarkson, 1981). Do the students prefer analytic approaches because the teaching is directed all the time to such an approach? Would students have a better grasp of mathematical concepts if visual approaches were given equal emphasis? A question a writer would have to ponder over.

Perhaps the third point following on from the first is for the power brokers rather than the curriculum developers. If teachers in the National High Schools are to be encouraged to help students to think through the mathematics presented rather than just teach the course, more thought needs to be given to who go to National High Schools. Although the year 10 mathematics score is at the moment a poor predictor of how students will perform at National High School in mathematics, it should improve with the new procedures to be implemented this year. However it is very doubtful that students with only a pass at year 10 mathematics will be able

to gain much from year 11 and 12 mathematics, and a student with an upper pass will have to work extremely hard. This is not an argument for an eletist school per se, such is against my egalitarian approach to things educational, but as argument for what these schools are meant to be; a preparation for higher studies.

These points will gain more credence if the suspected trends in this pilot analysis of results are confirmed after the full analysis is carried out.

NOTES

1. This research has been supported by a Papua New Guinea University of Technology Research Grant (No.232).

2. The originators of this project were P. Jones, C. Wilkins and H. Whippy of the Mathematics Department and P. Clarkson of the Mathematics Education Centre at Papua New Guinea University of Technology.

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AN EXTRANEOUS ROLE FOR THE VTR IN EDUCATIONAL RESEARCH

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Like any other educational media, the VTR, the latest innovation in instructional technology is devised primarily as an aid to the teaching/learning process. For example, a videotape on "The Derivative" might be produced with the aim of facilitating the learning of differentiation. Thus research on the VTR has been of the developmental and normative kind such as research to determine the feasibility of using the VTR in the classroom, or research to evaluate the effectiveness of the VTR as a teaching aid, or research to find ways of improving videotapes, ...

But, because the videotape provides a dynamic, permanent, and accurate record (both aural and visual) of a continuous sequence of events which vary over time - a characteristic of the teaching/learning process, the VTR is starting to assume a new role in education - aside from being the object of research it has become a tool for research. In particular, it has been showing its usefulness in the gathering of data in the research on the teaching and learning of mathematical problem solving at UNITECH.

QUALITATIVE VS QUANTITATIVE RESEARCH METHODS IN EDUCATION

The research in education before the decade of the 70's has been dominated by statistical studies such as:

1. testing the effectiveness of two drill strategies in facilitating the learning of basic multiplication

combinations which involved an analysis of covariance to test the equality of post test means;

2. analyzing the assumptions underlying in the cognitive domain by using "Scheffe's test of multiple comparisons" to evaluate significant differences between achievement levels;

3. develop instruments to assess the nature of student - teacher relations and classroom activities by using an analysis of variance to compare responses by two groups of teachers on an existing check list;

4. using the chi-square to determine if the teacher's experience is independent of their choice of teaching moves while engaged in the teaching of a concept.

This situation has been deplored by contemporary mathematics educators, among them Professor David Rappaport[1], who wrote,

> ".... the effort of statistical studies of global comparison is to lose sight of the child as an individual and to consider him only as an aggregate or collection. His score is thrown into the hopper of that important statistic - the mean."

However, due largely to the influence of the Piagetian School which looked at the learner as an individual, educational researchers have started turning to research methodologies where investigation of the subjects (teacher, students, or both) yields qualitative (such as descriptions of behaviour) rather than quantitative (such as scores, number of responses, etc.....) data.

These new methodologies can be classified into 3 general, not necessarily disjoint, types where observation is the common element. These are:

1. naturalistic observation which involves observation of teacher/learner behaviour in its natural setting with no interference on the part of the observer;

2. field studies which involve observation behaviour in its natural environment with the constraint that the experimenter has intervened in some way prior to the observation;

3. experimental studies in which the experimenter manipulates some aspect of the environment under controlled conditions and observes the effect of the manipulation.

Thus the reliability of the record of the observation will influence the validity and generalizability of the results of the research.

RECORDING THE OBSERVATION OF TEACHING/LEARNING BEHAVIOUR

The least occurate way to record the observation of a teaching/learning event is to rely on the memory of the observer. The most famous, if not the earliest example of such a record is the "Meno" written 23 hundred years ago by Plato where he described (with poetic, novelist and philosophical license, of course) how Socrates "taught" a slave a complicated geometric theorem.

Tapescripts from audiotapes could provide a more accurate record. In the two years of research[2] on problem solving by small groups at UNITECH, the audiotape recorder has been used to record the verbalizations of the problem solvers.

But the subjects did more than verbalize when they solved the problems - they drew diagrams, pointed to parts of the diagrams, made notes, performed calculations and measurements, did constructions with compass and straight edge, even counted on their fingers and toes (sometimes), consulted their notes, or the ceiling (also sometimes) .....

Hence, the audiotape scripts did not provide accurate records of the problem solving behaviour of the groups under study. On this third year of the research, the VTR is being used to record the problem solving behaviour of the groups as they solve problems in geometry. Analysis of data will start with watching the videotape recordings.

After all, if we want to find out what goes on in the pupils' heads when they solve problems we have to watch them solve problems.

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A PROBLEM SOLVING PROJECT
HOW TO DO IT

A Review of A Major
Problem Solving Project

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ABSTRACT

This paper reviews a major problem solving project from the USA, the Mathematical Problem Solving Project. The Project's aim was to improve the problem solving performance of children in grades 4-6. The emphasis was on developing the processes used to solve problems. Two approaches were used (1) the students were given a lot of problems to solve that were expected to elicit various useful processes of problem solving. They learned to solve problems by solving problems. (2) Lesson modules were developed to teach some processes (using tables, using guesses, and organizing lists) and problem decks were written to give students practice in using their new learned strategies. Both techniques proved to be successful.

The reports of the Project include extensive information on development and evaluation which I will touch on briefly.

The reports are valuable reading for one serious about problem solving. The review was presented at the Mathematics Education Conference, Lae, PNG; May 5-6, 1982.

A PROBLEM SOLVING PROJECT
HOW TO DO IT*

BACKGROUND

The Mathematical Problem Solving Project (MPSP) was a 2 year project in the mid '70's. Some of the names of the people involved will be familar to those of you involved in problem solving research: Frank Lester, Jr. ,John LeBlanc, Norman Webb, George Immerzeel and Jeremy Kilpatrick. The project involved staff at 2 universities and one school district in 3 American states (Indiana, Iowa, and Michigan). This was an unusual organizational form, common in business but not in educational research projects. The Project was productive in the years just before the Euclid Conference and the National Conference of Supervisors of Mathematics statements about basic skills and problem solving. These statements have directed teachers' thinking and objectives for the 1980's. The MPSP may have had some influence on the participants at Euclid and members of NCSM.

I was interested in reviewing this project because it was one of the major problem solving projects. It was well done and well documented. The final report of May 1977 had over 1500 pages on project activities and materials that were developed and evaluated.[1]

The project was a major development project and I would recommend this final report to anyone wishing to set up a project to develop instructional materials in mathematics, especially in problem solving. It is a good "How to" of materials development projects.

INTRODUCTION

Some people would say that the most important goal of school mathematics is to develop the ability to solve problems. Recently there has been an upsurge in interest in problem solving instruction and research. There is a need for systematic approaches to instruction in problem solving.

Before discussing the Project and problem solving further I think some discriminations must be made.

* I acknowledge the support of the Goroka Teachers College Research Committee as their funding was used to purchase some of the materials from the Project's Final Report.

1 Most of these papers are available as ERIC documents ED168833 -ED168856.

What is a problem?

The project defined a problem as a situation in which an individual or group was called upon to perform a task for which there was no readily accessible algorithm which determined completely the method of solution. You could say the path to the desired goal was blocked. The problem must be interesting to motivate the solver to seek a strategy to find the solution.

A problem is a very individual thing. A problem for one student maybe an exercise for another.

In psychological terms, a problem is a stimulus response for which an organism does not have a ready response.

Problem Solving

Problem solving refers to the behavior of an individual in a situation in which previous experience, knowledge and intuition must be coordinated in an effort to reach a goal for which a procedure is not known.

The Project discriminated between the typical textbook problems and the process problems in which the Project was interested. Textbook problems are designed to elicit the use of one or more of the 4 basic operations and usually follow a section of the text that guides solution. The student can directly apply an algorithm to reach solution.

> Apples cost 10 t. John buys 3 apples and Kiki buys 5 apples. How much did they spend together?

Process problems cannot be solved using algorithms known to the students but require one or more of the general problem solving strategies.

> I have K1.60 in coins. This is made up of 20 coins. If they are 5t and 20 t coins. How many of each are there?

It was felt that the process problems held the greatest potential for eliciting problem solving in its broader sense and for eliciting strategies.

The Project was interested in the process problems. Materials were developed to get students to become more aware of the problem solving processes and to provide more strategies for use when faced with a problem. The materials consisted of teacher's guides for lessons, student lesson materials and problem decks for 3 problem

solving strategies.

Organizing a list
Using guesses and estimates
Using tables

Other discriminations could have been made beside that between process and routine textbook type problems. One could discriminate between problems for which the solution is to be found to a high level of precision or for which the solution need not be,and perhaps cannot be very accurate.

One could discriminate between problems that require further information than stated in the problem and those that contain all required information except the solution process. Many real-world problems require the solver to gather additional information through experimentation, inquiry, measurement, etc. The Project staff saw these "real-world" problems as important and developed a few (See miniprojects in Appendix E).

Discriminations could be made between problems to find a solution (unknown) and problems to prove, through logic, the truth or falsehood of a situation.

Project Goals and Strategies

The overall goal of the Project was to improve the problem--solving performance of children in grades 4,5,& 6. The emphasis was on improving the processes used in solving different kinds of problems.

More specifically the Project intended to:

1) Conceptualize a model of the problem solving process.
2) Develop instructional materials and techniques consistent with the problem-solving model and to engage classroom teachers in the development and trial of these materials.
3) Evaluate the effect of these materials on the problem-solving performance of children.
4) Collect and organize observations and conjectures related to children's problem-solving activities.
5) Design and carry out a dissemination model.
6) Explore the use of the hand calculator in child problem solving.
7) Explore the possibility of developing appropriate problems which use real applications of mathematics.

The project was to have a 4-5 year life but US goverment funding was cut after 2 years so some of the goals of the project were not reached.[2]

1) A Problem-Solving Model

A search of problem-solving literature helped form the basis for a conjectured model of the problem solving process. It consisted of 6 stages:

Problem awareness
Problem comprehension
Goals and analysis
Plan development
Plan implementation
Procedures and solution evaluation.

It is pointed out that these are not necessarily sequential.

Problem Awareness

A problem is posed and the student must see it as a problem, not readily solved. This recognition may come after several attempts to obtain the goal have failed. A component of this stage is motivation to do the problem once it is recognized as a problem.

Problem Comprehension

The student must translate the problem information into terms which have meaning for the student. The student must internalize the problem, sorting out the relevant information and determine how it interrelates. The result is an internal representation of the problem which may not be as accurate as necessary to solve the problem and maybe adjusted as the student progresses toward a solution.

Goal Analysis

The internal representation of the problem furnished the student with a means of establishing goals as priorities for working on the problem. At this stage the student may establish subgoals to aid in problem comprehension and development of procedures. The student may reformulate the problem so that familiar strategies and techniques can be used.

2 The project was successful at isolating the organizational and funding difficulties of the Project from the main areas of interest. Other educational researchers could learn from the Project's efforts in this area.

Plan Development

There needs to be a conscious attempt to devise a plan of attack. Potential strategies must be identified, like finding a pattern or solving a simplified version of the problem. This is the most interesting and least understood stage. As one watches a person solving a problem you might think "How did s(he) ever think of that?"

A source of difficulty at this stage is that students give up if the plan of attack doesn't come easily. A "good" problem will lead to failures initially. Perhaps students don't have many plans at their disposal to find a good one that can be successful. Grade 4-6 students tended to use trial and error or give up (a strategy that gets the teacher to tell them the answer; the goal is achieved). The MPSP looked at teaching strategies and helping students organize their thinking to aid in plan development.

Plan Implementation

In this stage the student tries out the devised plan. Some executive errors may occur to interfer with a good strategy (computation errors or carelessness). If the plans are not based on well established strategies,the student may become confused during implementation or may forget what s(he) is doing.

Procedures and Solution Evaluation

Evaluation should be going on during all stages of the solving process. Evaluation takes the form of questions asked at all stages, ie. what data in the problem is relevant to the problem's solution,have I identified the conditions operating in the problem, have I found the best way to do the problem, is the problem a version of one that I have solved before, am I using the strategy correctly, does my solution satisfy the conditions of the problem? The solver should store what was learned in solving the problem so s(he) can use it later.

I would like to sneak in an idea here. A further step might be "Reaching the Goal". This is to emphasize that solution evaluation might not mean that the goal has been reached. It is an important step in any successful problem solving task before the goal is reached.

This model provided a framework to guide the project activities. It helps identify those factors that might have the most influence on problem solving success. It is possibly incomplete as it may not

account for all types of problems. It maybe that there is no single model that can accurately depict problem solving for all types of problems or all problem solvers.

A model like this can also provide teachers with a framework for organizing problem-solving instruction. They can see it as a development of processes rather than just presenting isolated tasks. Some Project staff suggested that 2 models were needed: a descriptive model of the problem solving processes and a second one to guide instructional intervention in problem solving (instructional model).

While observing students as they solved problems, the project staff identified 3 areas of difficulty for students in the problem solving process.

--Students misread or misinterpreted problems.[3]

--Students had difficulty retaining or coordinating multiple conditions in a problem.[4]

These difficulties are at the problem comprehension stage and are areas of needed research.

--Students do not appear to use any strategies during problem solving.

This is a difficulty at the plan development and implementation stage of the model. The Project concentrated on this stage of the problem solving process.

2) Developing Instructional Materials

This is the area in which the Project staff devoted the most time. It was felt that research verification of generalizations concerning child problem-solving would be years in coming. The staff had considerable experience in child problem-solving so they decided to begin materials development in the early stages of the project. Two instructional strategies were to be used. One was to

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3 Students often perceive a simplified version of a stated problem. Students then proceed to solve the perceived problem sometimes not even seeing that a problem exists due to misinterpretation or simplification because students can't understand words or phrases. Factors that influence problem comprehension need special attention. As most problems are presented in written forms attention is need to reading difficulties (readability).

4 There is a need to investigate how the number of conditions in a problem influences success and what are important factors of this aspect of problem solving.

teach identified skills and processes which are useful in solving problems. The other was to provide problem-solving experiences and then look back on the experiences and analyze the skills and processes used.

The modules were developed to teach specific skills and processes (using a guess, making a table, and organizing a list) as per the first instructional strategy. These will be discussed later. (see Appendix A)

The second instructional strategy was supported with materials in the form of a Problem-Solving Bulletin Board. This will also be discussed later. (See appendix B)

The materials were generally enjoyed by children. The teachers saw evidence of growth in attitude and confidence in problem-solving tasks. The materials were effective in developing processes which seem important for problem-solving.

3. Evaluation of Materials

Information was collected during module trials in order to aid in the development of the modules. This formative evaluation took the form of quizzes, record of time spent on modules, number of cards completed in each deck, interviews of teachers, etc.

Summative evaluation was by 2 standard instruments used to measure problem-solving achievement. Also used was a problem-solving survey and an attitude questionaire developed by the Project. Finding suitable standard measuring instruments was complicated by the fact that the MPSP problems were not of the type used in math problem-solving achievement tests.

An instrument for measuring specific problem-solving processes was being developed but was not completed by the end of the project. See appendix C.

Evaluation was also made of teacher views of the type of problems that are appropriate for children and are enjoyed by them. A problem-sort task (appendix D) was used for this.
Teachers' perceptions of the type of problems appropriate for and liked by children changed as the teachers used the project modules. This change was towards the MPSP type problems.

4. Observe Children Solving Problems and Development of Conjectures

The following observations were made of children (grade 5)

in problem-solving situations:

They usually wanted to "compute" answers. The children were conditioned to the 4 basic computational operations.
They were answer oriented.
They didn't want to "mess around" with paper and pencil.
They were not used to working in small groups cooperatively.
They had not had experience with process-type problems.
They relied heavily on adult (teacher) guidance and direction.

With some encouragement,the children began to solve process problems in the early problem sessions of the project.

A list of conjectures related to problem solving was begun (see appendix E). Important conjectures related to students in unguided experiences in problem solving. This will be discussed in a later section on learning to do problems by doing problems.

5. Design a Dissemination Model

Two dissemination models were utilized. One was informal presentation of materials at meetings. The other kind was more formal planned inservices with teachers who planned to implement project materials. Classroom observations were made to gauge how much inservice on MPSP goals and materials was needed to successfully implement the materials. The funding was cut before completion of this phase.

6. Use of hand calculators

MPSP viewed the hand calculator as a tool that could be useful in solving certain kinds of problems. Its advantage was in making large messy numbers easy for kids to handle and also make more children willing to experiment with problem solutions. Extensive work was done at the University of Northern Iowa with children using calculators. I don't have this work, but I believe it was published elsewhere.

7 Develop Appropriate Real World Application Problems

The problems of the MPSP modules were chosen because they emphasized the use of certain processes which were to be developed in children. This is different from the standard text book exercises meant to develop skills with certain algorithms.

It is recognized that a third type of problem is needed for children; problems to bridge the gap to real world problems from the process and skill type problems. The problem needs to develop skills with problem definition, with mathematical modeling, with data collection and with answer interpretation. These problems are very difficult to develop for elementary school children. The MPSP developed some called miniprojects (see appendix F).

INSTRUCTION MODULES

Various skills and strategies were identified for use with students in grades 4-6:

Using an equation
Using a table
Using resources
Using a model (physical, graph, picture, diagram)
Making a simpler problem
Guess and test (estimate)
Compute to solve
Organzing a list

The project settled on 3 strategies on which to focus development of instruction modules:

Organizing lists
Using guesses
Using tables

Each module consisted of a student instruction booklet with teacher's guide, students' problem deck and teacher's decks. The modules were developed, trialed, revised, tried,etc. Examples of each component are shown in appendix A.

Organizing lists module consisted of 8 lessons with the following goals: reading and organizing lists, making headings and entries for lists, solving problems where the list is the solution and problems where the solution is contained in the list, performing calculations on list entries to solve, solving problems when the list suggests relationships that help solutions,and solving problems using tree diagrams. There were also 58 problems at 3 levels that could be solved with lists,and various forms of quizzes.

Using guesses module consisted of 5 lessons aimed at these goals: using guesses and computation to solve problems, using

guesses to solve two-step problems, using guesses and diagrams to solve problems, using estimation to solve measurement problems, and using guesses to solve problems. There was a problem deck of 125 problems at 5 levels of difficulty, plus quizzes.

Using tables module consisted of 6 lessons to develop these skills: making a table, completing a table, using tables to solve problems, and reading tables. There was a problem deck of 150 problems at 5 levels. Four types of tables were used: those with a constant sum, constant difference, constant ratio or quotient, and constant product. There were also quizzes.

The project funds were cut short and so there was not a full summative evaluation of the modules. Instruments were developed and used during formative trials of the materials. Trends generally favored the experimental students on solving of process type problems and equalled or bettered the control students on traditional type problems. Results related to student attitudes were not clear.

LEARNING TO SOLVE PROBLEMS BY SOLVING PROBLEMS
(Learning by Doing)

A very interesting aspect of this project was the work on helping children to develop problem solving ability without formal instructional intervention. Staff of MPSP developed a problem-solving Bulletin Board. (See appendix B) The board consisted of 3 to 4 parts depending on the stage in the project. The problem was displayed along with some work from other child. Some questions were posted to act as hints. Some extension of the original problems was posted also but later dropped because teachers did not know how to use it. (They didn't feel comfortable with it). At first answers were not supplied to teachers but later they were supplied to help the teachers feel more comfortable. Some teachers reported that they didn't look at them before posing the problem to the class so they could honestly tell students that they didn't know the answer and could encourage students to work together toward solutions.

After a short exposure to the Bulletin Board, the students stopped asking questions of the teacher and showed a marked increase in the uses of other processes to solve problems.

Project staff attempted to make the problem tasks interesting and familiar in context. The computational requirements of the tasks were minimal so success was likely. Various methods

could be used to solve the problems. After solving sessions, the groups could share their various methods. Many of the problems had multiple conditions that had to be considered simultaneously. Students seems to like these problems.

To gauge student growth in problem solving ability, equivalent problems were presented at an interval following the first presentation of a problem. The single exposure to the first problem of the pair caused the second to be seen as an exercise because skills were now available to solve it. The second one was "too easy ".

Although the project didn't run its expected life, it was very productive and answered a lot of questions. Many more questions were raised but at a deeper level of understanding.

The project reports are an excellent guide book for those interested in developing projects in problem solving with children.

Project reports included in Final Report:

- Review of the literature
- Problem Solving in the Elementary School-
  - Some Education and Psychology Considerations
- You Can Teach Problem Solving
- Learning to Solve Problems by Solving Problems
  - Plus Instructional Materials
- Module Development and Formative Evaluation
  - Plus Evaluation Instruments
- Module on Organizing Lists
  - Lessons and Problem Deck
- Module on Using Guesses to Solve Problems
  - Lessons and Problem Deck
- Module on Using Tables to Solve Problems
  - Lessons and Problem Deck
- Summative Evaluation and Instruments
- Process Evaluation
- Problem Sort Tasks for Teachers
- Miniprojects
- Overview

-- Appendix A --

PROBLEMS FROM ORGANIZING LISTS
LESSONS AND PROBLEM DECK

1) List the ways to arrange the letters of the word "EAT".

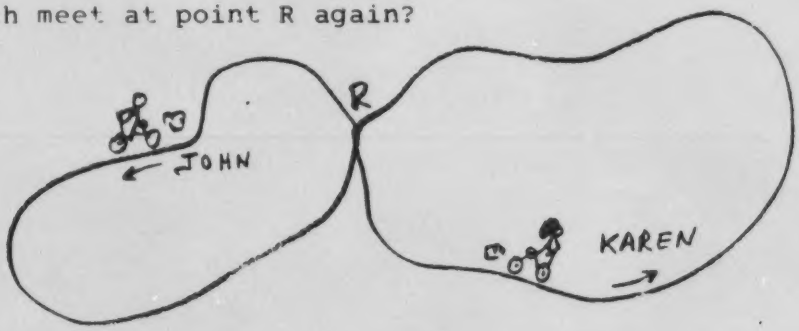
   a. How many ways can the letters be arranged?
   b. How many of the new words are real ones found in the dictionary?

2) Ask 10 of your classmates for some information: their name, their age, birthdate, favorite singer, which school subject they like most, and their favorite sport.

   Make an organized list of the information.

   a. Who is the oldest?
   b. Who is 6th oldest?
   c. Who are the 3 youngest girls?
   d. Which subject is liked by the most students?
   e. Did one singer get the most votes?
   f. What did you find out about favorite sports?

3) Behind John and Karen's house are two mini-bike tracks. It takes John 12 seconds to complete each lap. Karen completes a lap every 15 seconds. If they both start at point R at the same time, how many seconds will it be before they both meet at point R again?



-- Appendix A --

PROBLEMS FROM USING GUESSES
LESSONS AND PROBLEM DECK

1) Lonny has 2 bats and 1 baseball. They cost $11 together. Melanie has 1 bat and 2 baseballs. They cost $7 together.

How much does 1 bat cost?
How much does 1 baseball cost?

2) This page is 29 cm long. Estimate its width.

3) A full size bathtub holds about 45 gallons of water.

1. Estimate how much water you use when you take a bath.
2. Estimate how many baths you take in a year.
3. Estimate how many gallons of bath water you use in a year.

4) Your teacher gives you 1 bonus point on the first day of school. 2 bonus points on the second day. 4 bonus points on the third. 8 bonus points on the fourth day and so on. Which of the numbers below tells you how many bonus points you will get on the twelfth day?

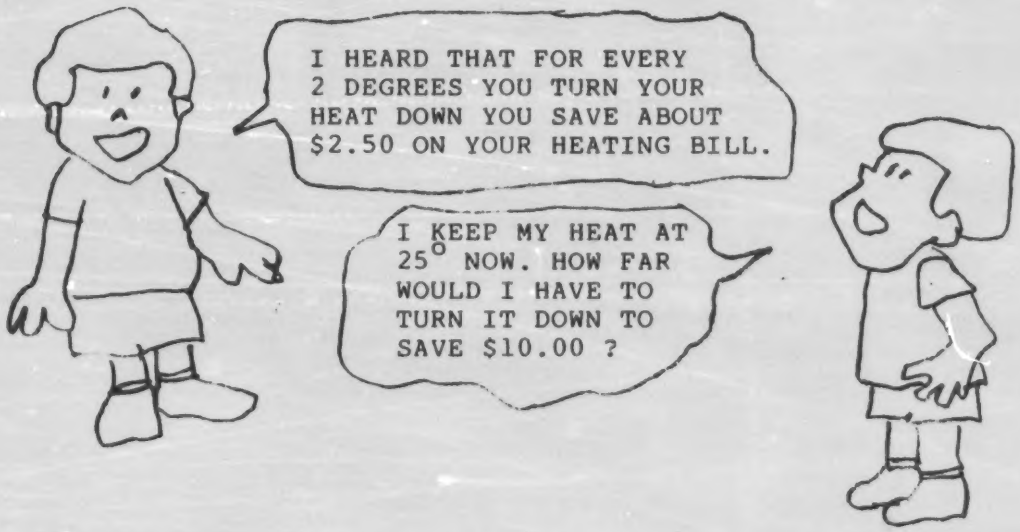
128 256 1024 2048 4096

-- Appendix A --

PROBLEMS FROM USING TABLES

LESSONS AND PROBLEM DECK

1)


I HEARD THAT FOR EVERY 2 DEGREES YOU TURN YOUR HEAT DOWN YOU SAVE ABOUT $2.50 ON YOUR HEATING BILL.
I KEEP MY HEAT AT 25° NOW. HOW FAR WOULD I HAVE TO TURN IT DOWN TO SAVE $10.00 ?

2) When you listen to a cassette tape the counter moves from 0 to 36. How long has the tape played as the counter moves from 0 to 288 ?

FILL IN THE TABLE

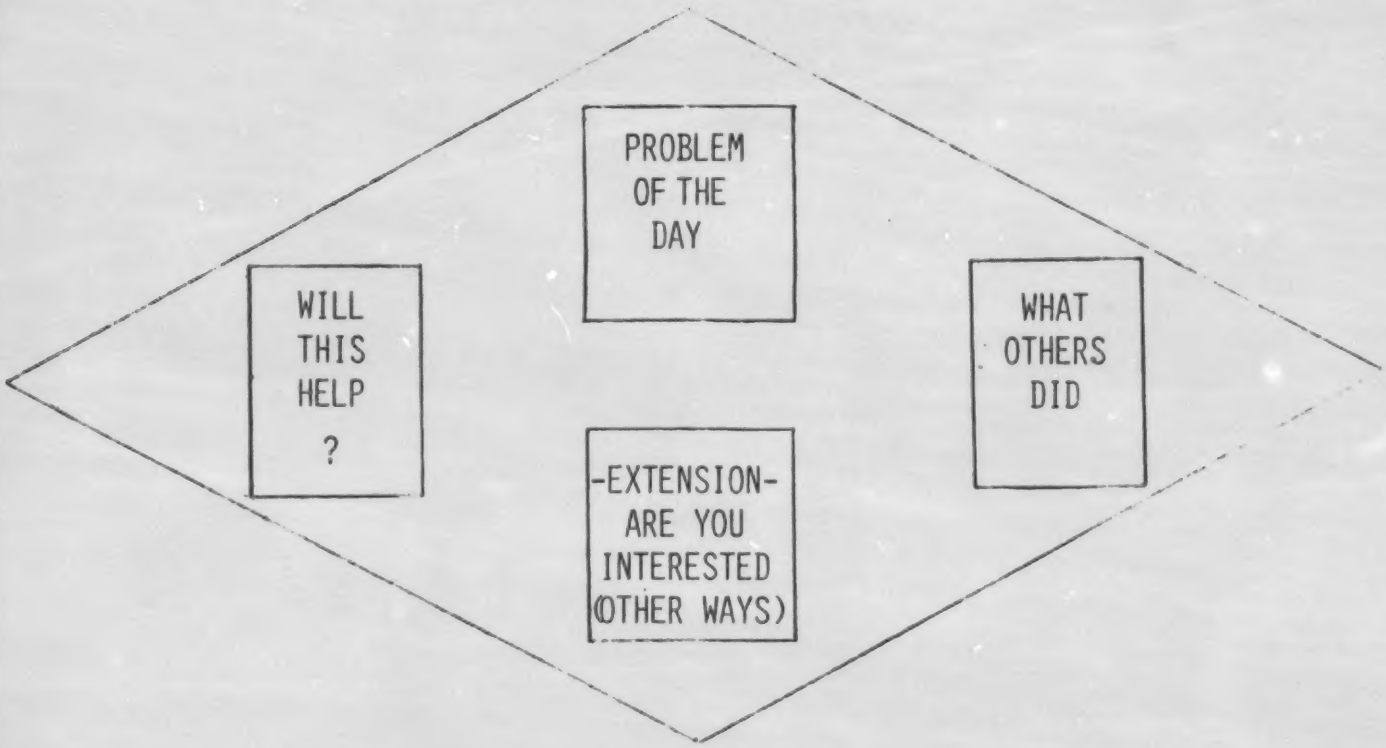
Minutes												
Counter reading												

How long does the tape play as the counter moves from 0 to 540 ?

3) A ferris wheel takes a group of 11 people every 5 minutes. How long will you have to wait in line if there are 85 people in front of you ?

-- Appendix B --

PROBLEM SOLVING BULLETIN BOARD



PROBLEM INTRODUCED	SOLUTION EFFORT	PROBLEM DISCUSSION
Students will understand problem statement	Students will develop and try at least one strategy for solving problem	Students will discuss their respective efforts and gain new insights into the problem

--Appendix B--

THE PROBLEM

Jim works in an ice cream store. Ice cream cones cost 25cents. Show the ways that Jim can be paid exactly 25 cents for an ice cream cone.

WILL THIS HELP?

1. What are the different kinds od coins that can be used to solve this problem?
2. Can you guess how many ways there are to make change for a quarter?
3. Can you use 3 dimes to make change for 25 cents?
4. Would it help to put the answers you've found in a table like this?

Quarter	Dime	Nickel	Penny	Total
0	1	2	5	25

5. Why are you interested in this problem?
6. If you had 3 nickels, what are the 2 ways you could make 25 cents? (One way uses dimes. The other way uses pennies.)

HERE'S SOME MORE....INTERESTED?

1. If a sherbert cone costs 20 cents, show the ways Jim can be paid.
2. If two dips cost 35 cents, show the ways Jim can be paid.
3. A man came into Jim's store and spent 75 cents. If a soft drink costs 10 cents, show what the man could have bought for his 75 cents.

   Remember: Single dip 25 cents
   Double dip 35 cents
   Sherbet 20 cents
   Soft drink 10 cents

4. Make up a problem like this one and give it to a friend to do. (You might want to try it at home, too!)

One way to answer the "change for 25 cent" problem is:

Quarter	Dime	Nickel	Penny	Total
1	0	0	0	25
	2	1	0	25
	2	0	5	25
	1	3	0	25
	1	2	5	25
	1	1	10	25
	1	0	15	25
		5	0	25
		4	5	25
		3	10	25
		2	15	25
		1	20	25
			25	25

The children's work will probably not look like this. That's OK.

****

REMEMBER

1. Make sure everyone understands the problem.
2. Working on the problem is more important than getting a solution. Let the children "mess" with the problem.
3. Encourage children to talk with each other and work together if they want.
4. Allow and encourage as much use of the bulletin board as possible.

****

-- Appendix C --

Process Evaluation

A CARERPILLAR IS PUT ON THE BOTTOM OF A JAR THAT IS 8 INCHES HIGH AND 7 INCHES ACROSS. THE JAR HAS A LID WITH HOLES IN IT. EVERY DAY THE CATERPILLAR CRAWLS UP 4 INCHES. EVERY NIGHT IT SLIPS BACK 2 INCHES. HOW MANY DAYS WILL IT TAKE FOR THE CATERPILLAR TO TOUCH THE LID OF THE JAR ?

Check the information that helped you solve the problem.

Check what you think the problem asked.

What did you do to get the answer ?

Here is what some people do. Make a check next to each one you did while working the problem.

What did you do as soon as you read the problem ?

Did you check your answer when finished ? Why ?

The questions above were asked of the students after solving the question. Prompts were given which were to be checked.

-- Appendix D --

PROBLEM SORT TASKS FOR TEACHERS

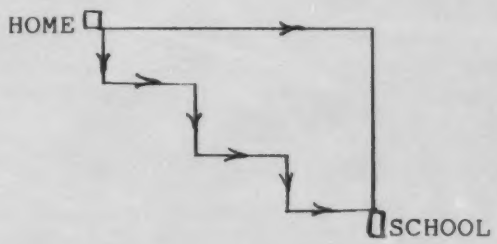
Teachers were asked to indicate which of a set of 24 problems they would use and which they would not use to teach problem solving. They were also asked which ones children would be interested in solving.

Sample problems from the set are shown below.

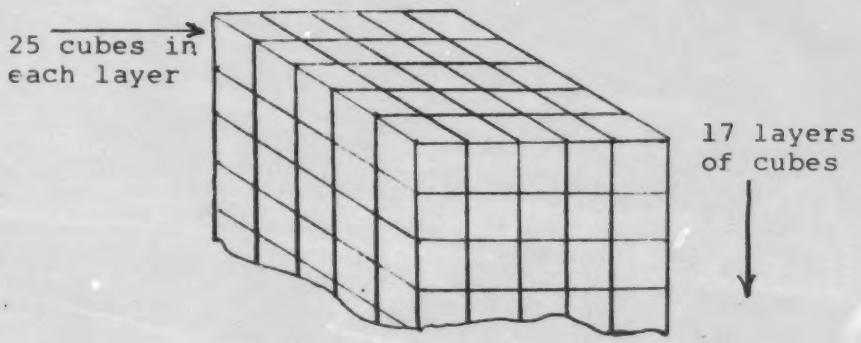
1) 87 ÷ 3 How many threes in 87 ?

128 ÷ 4 How many fours in 128 ?

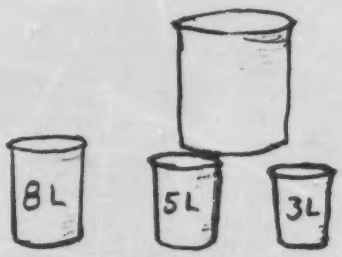
2) Susan and her sister Sara walk to school each day. Sara knows a short cut to school. If the girls start at the same time and walk at the same speed, who will get to school first? Explain your answer.



3) Give the volume.



4) How can you get 4 litres of soft drink from a large container if you have only containers that hold 3, 5, and 8 litres? (You can't pour any back)



--Appendix E--

Conjectures:

Children can solve process problems, but their strategies are limited to trial and error.

Children have difficulty with multiple condition problems.

Children can be taught some problem-solving strategies that can be used to solve new problems.

An effective way for children to learn to solve problems is by solving problems. See "Bulletin Board" described in Appendix B.

The acquisition of problem-solving strategies by children seems to be related to teacher directed analysis and generalization of problem solving processes.

MINIPROJECTS

AN AVERAGE SIZE TREE WILL PRODUCE ABOUT 25 KILOGRAMS OF PAPER. ESTIMATE HOW MANY TREES MUST BE CUT DOWN TO PRODUCE ALL THE PAPER USED BY YOUR SCHOOL IN ONE YEAR.

Miniprojects are meant to meet the following needs:

They offer the students an opportunity to use their mathematical skills in situations which are similar to real-world problems. They show that mathematics is useful.

They are designed to give the students introductory experiences in elementary mathematical modelling, i.e., using the scientific method to handle real-world problems.

Many of the projects give the students experience in collecting, handling, and interpreting data; they provide experiences in organizing information.

The projects give the students practice using measurements or numbers with units in a natural setting. Their calculations often involve estimating and rounding off. The children must decide which operations must be used to achieve a desired goal.

The miniprojects provide an opportunity for the students to engage in an activity which is similar in character to applications of mathematics that one encounters in later life. It teaches them how to handle such situations independently and should enable them to use their mathematics more effectively as adults.

Many of the projects give the children practice in taking a large problem, breaking it down into smaller problems, and setting subgoals with the aim of progressing towards the final goal.

"IN ALGEBRA WE DO OPERATIONS ON LETTERS"

By
Harold R. Frederick
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Goroka, EHP

Paper presented at the Second Mathematics Education Conference. Lae, Papua New Guinea. May 5-6 , 1982.

ABSTRACT

This paper is intended to lead to discussion of an investigation into some common errors that students make when they translate an english language representation of a relationship into an algebraic representation. It presents the idea that students use of a letter as a label and the syntactic vs semantic translation methods and passive vs operational idea of equality may be possible sources of error.

Students at GTC and PNGUT have responded to test items written to elicit their translation skills. One group was tape recorded to evaluate their strategies and the use of open, small group discussion as a data producing method. Tentative thoughts are presented about the errors found.

The outcome of the investigation will be the development of methods to introduce the beginning algebra topics in grade 9 and 10 of the provincial high schools. A good start toward this goal has been started by Mike Siford (1981).

IN ALGEBRA WE DO OPERATIONS ON LETTERS*

By
Harold R. Frederick
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Goroka, PNG
May 1982

BACKGROUND

In high schools in PNG, the understanding of algebra as generalized arithmetic in an important objective. Students going on to higher levels of mathematics must have a thorough and well based understanding of generalized arithmetic (algebra). Support for algebra work in the high schools comes from Subject Masters and teachers in the Universities and other tertiary institutions (Siford, 1981).

A number of projects have been looking at algebra and high school students' understanding of the topic (Galvin,W. & Bell,A., 1977; Kuchemann,D., 1981; and Clement,J., Lochhead,J. & Monk,G., 1981). Scores on tests given in these projects show that students have a poor grasp of some algebraic manipulations and a poor grasp of the concepts of algebra. Kuchemann (1981) discriminated between 6 different uses of letters by students on an algebra test. The 6 levels at which letters were used are:

Letters evaluated
Letters ignored
Letters as objects
Letters as specific unknown
Letters as generalized number
Letters as variable

A very common use of letters that appears to cause many of the errors on the test was the use of a letter as an object (label for an object).

Members of the Cognitive Development Project (University of Massachusette, USA) are looking at students' understanding

* In arithmetic we do operations on numbers..... . Comment from teacher. Source unknown.

of letters in algebra as one path to help them to understand some of the difficulties some of their Physics and Engineering students have with algebra (Clement, et al, 1981; Rosnick,P., 1981).

STIMULUS AND THOUGHTS

Engineering students at the calculus level were asked to respond to problems like:

> Write an equation to represent the following statement: "There are 8 times as many people in China as there are in England." Let C be the number of people in China and E be the number of people in England.

Translation of the sentence into an algebraic sentence was incorrectly done by 30%- 40% of the students. How can this be with students who at the same time can find the derivative of $f(X) = \sqrt{X^2 + 1}$. Kuchemann (1981) gives us a hint. The student may be using the letters as labels for the groups in the problem. Clement (1980) analyzes this idea further. From protocol analysis he sees three types of translation processes. The simplest one leading to the wrong answer (8C = E) is a word order "syntactic translation". The student simply directly maps the information in the sentence onto the equation, retaining word order. This performance requires little or no understanding of the original sentence. The second method of translation is a "semantic translation" method. The student shows an understanding of the relationship between the sizes of the proups but simply uses letters and numbers to label the groups (8C = E). S(he) lacks understanding of the equation that must be generated but understands the meaning of the sentence.

Correct answers come from students who seem to understand the operational equality of the equations. They see that they must increase the number of people in England 8-fold to match the number in China and they show it in the equation (C = 8E).

The results of other problems where translation was required were the same or worse. Only 42% of first - year engineering

students (n = 381) got this right:

> Weights are hung on the end of a spring and the stretch of the spring in measured. The data are shown on the table:
>
> | stretch S (cm) | weight W (g) |
> |---|---|
> | 3 | 100 |
> | 6 | 200 |
> | 9 | 300 |
> | 12 | 400 |
>
> Write an equation that will allow you to predict the stretch (S) given the weight (W).
> (Common error: 3S = 100W)

A startling result comes from this problem (Rosnick,P.,1981) which shows again the use of letters to stand for objects rather than the number of the things:

> At this University, there are six times as many students as professors. This fact is represented by the equation S = 6P.
>
> A) In this equation what does the letter P stand for?
>
> 1) Professors
> 2) Professor
> 3) Number of professors
> 4) None of the above
> 5) More than one of the above (which ones)
> 6) Don't know
>
> B) What does the letter S stand for?
>
> 1) Professor
> 2) Student
> 3) Students
> 4) Number of students
> 5) None of the above
> 6) More than one of the above (which ones)
> 7) Don't know

About 40% of the 152 students doing this problem were not able to chose the correct "number of professors" for part A and 43% were not able to answer B correctly. What are your thoughts about the distractors in part B? "S stands for professor"? Well, about 22% chose this as the answer. Every one of them also chose "none of the above" for part A of the question. This is a consistent response for those students who view the

letters as objects.

Last year and several times this year, tests similar to that attached have been given to Goroka Teachers College students and a small number of University of Technology students. The errors that show up are the same as in the overseas research. You might be interested in the results on some questions from the Unitech sample.

n = 11 (total percent may not equal 100% as other responses did not fit these classifications)

QUESTION →	3	7	8	9
CORRECT	55%	36%	18%	18%
INCORRECT	45%	55%	55%	55%

Discussion of this outcome might be immediatly valuable in helping these students with their difficulty as PNGUT and GTC lecturers become aware of the frequency of the errors. Potential methods of remediating the difficulties may be discussed.

FURTHER DISCUSSION

In arithmetic and other high school topics one can be taught a procedure, practice it, and just do it and know it is right. It seems though, in algebra there are not always easy procedures to get the answers. More understanding is needed and a checking process is needed at the end. This may be the "critic" of the information processing research or the executive control processes. Perhaps the "critics" are what determine success or failure with the type of translation problems I am discussing. Now the question is, how do we train these "critics"?

Inquiries will now take the form of interviews with students of varied mathematical background who show the different translation strategies and skills in an attempt to find the bases of the errors. I will then attempt to alter the students' translation strategies in the interview session or in small group teaching. This will hopefully give me some idea of the translation process. Through the interviews I would hope to gain insight into the language difficulties that students might have with the english sentences used. This would of course lead to translation difficulties. With knowledge about the source of the errors I can then develop a teaching strategy to help students to gain a broader understanding of variable and to understand the relationship between the representation of statements in algebra and the representation of the statements in english.

A thought to end with. One of the first subtopics of early algebra work is simplifying by combining like terms. The way it is frequently taught may interfere with full development of the variable concept. Out of expediancy the topic is introduced with letters standing for objects. This makes it easy for grade 9 students to pick up the skill of combining like terms but it does nothing to develop the variable concept of letter use in its broader sense. Mike Siford (1981) has presented some valuable activities to help teachers with the topics in algebra.

How many of your students would say "S stands for professor"?

AND FURTHER DISCUESSION

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THE CASE FOR CHANGES IN APPROACHES TO MATHEMATICS CURRICULA : SOME SUGGESTIONS FOR RESEARCH AND TEACHING.

John Gough
Lecturer in Mathematics Education
University of Papua New Guinea

A paper presented at the Second Mathematics Education Conference, Mathematics Education Centre, Lae, 1982

Part One : THE REASONS WE NEED TO CHANGE.

Recent research in several different areas of education theory leads to the clear conclusion that current text-book approaches to mathematics education at secondary and tertiary levels need to be changed. Such changes will stress the *meaningfulness* of the mathematics concepts being learned. They will place the concepts into a *wider context* of social and real-world significance, and they will make the learning of mathematics more effective by fitting better the *learning style* pupils use.

(1) Research in Philosophy of Mathematics.

The work of Imre Lakatos presents a view of the way a mathematician works which makes mathematics research look the same as science research. Lakatos proposes a problem-solving view of mathematical discovery which is more general than, but can contain the specific practical techniques described by George Polya.

According to Lakatos a mathematician attempts to solve a problem by proposing *conjectures* , or hypothetical concepts or solutions. These conjectures are then *criticised* and *tested* by analysis and counter-examples and attempted proofs. The conjecture is compared to other, possibly related concepts, in order to clarify the possible meaning of the conjecture. Thus mathematical discovery is seen to be a process of meaning-making.

This view stands in marked contrast to *deductivism*, the usual text-book approach of definition, axioms, proofs and theorems. Lakatos's theory, and discussion is supported by entertaining case studies in the history of mathematics, particularly dealing with the problems of finding regular geometric solids or polyhedra - how many polyhedra are there? how can we prove there are no more? what *is* a regular polyhedron? Lakatos's theory is compatible with Karl Popper's theory of Scientific Method. It is also very similar to recent theories of the *psychology of learning*, such as that of Richard Skemp. It begins to seem that an individual *learns* in a way which is similar to the way mathematicians *make* new mathematics: by a kind of experiment.

(ii) Research in Curriculum Theory.

Philosophy of curriculum has for many years been dominated by the theories of Paul Hirst who has argued that there are seven distinct *forms of knowledge*. These forms are distinguishable by their *subject matter* and also by their *criterion for determining truths* within the curriculum area.

According to Hirst *mathematics* is one of these distinct forms of knowledge. He argues that it needs to be taught in ways which are different from teaching methods for other forms of knowledge.

This view is seriously questioned by Lakatos's philosophy of mathematics, and other philosophers of curriculum offer similar criticisms.

Kenneth Ruthven argues that mathematics is in fact *indistinguishable from science*. Ruthven says that it needs to be taught within a clear cultural perspective, seeing mathematics as a socio-historical phenomenon. This is in order to make clear the method of informal discovery and criticism by which mathematics is *invented* and *learned*. Jere Confrey offers similar arguments, calling for teaching mathematics in an across-the-curriculum way which can better reflect the nature of mathematics. Such an approach also stresses the essential link between knowing and learning, which we saw was suggested by Lakatos's theory.

(iii) Research in Psychology of Learning.

Recent work in Cognitive Psychology and Information Processing, as well as research into the way very young children learn to speak their mother tongue have begun to show that *meaning-making* is central to learning. That is, a person learning a new idea begins by realising that there is something he does not know, and, faced by this problem, he attempts to learn by forming a *hypothesis* or rough mental working model of the new idea. He then tries to fit this hypothesis against things he already confidently knows, and compares this with evidence he has to hand which seems relevant to the new idea. Then by a process of *criticism*, or *feedback* (both positive and negative), and *error-elimination*, and *concept-modification* he arrives at the situation of feeling that he has *understood* the new idea, or *made sense* of it.

Such a meaning-making approach to the psychology of learning is compatible with, but goes beyond the early theories of Piaget, Ausubel,

Bruner and other cognitivists. For example, Piaget talks about a person learning by a combination of the processes of *assimilation* (in which new and puzzling information is seen to be like things which are already known) and *accomodation* (in which new and puzzling information is understood by *new* ideas being invented by the learner). These processes are really another way of describing meaning-making by a process of critical error-elimination and concept modification - hypothesis forming, testing and adjusting.

It is compatible with computer-based theories of psychology. Richard Skemp takes such an approach to the psychology of learning mathematics, as does Sylvia Farnham-Diggory and the Russian psychologist Krutetskii.

(iv) Where To, Then ?

These critical views of standard mathematics curricula lead to a need to change from dry, deductivist *text-book* approaches, through definitions and exercises, and the learning of relatively limited and *isolated "recipes"* for problem solving. The usual deductivist approach is to define the terms and rules being discussed. Then to explain the results or theorems that we can derive. Then one or two examples are given of how these theorems are *used* - this is the *"recipe"*. Then practice exercises are given to "learn" the recipe. Later in an exam, the student's task is to *identify which recipe fits* which question, then plug in the numbers and do it.

We need to introduce a new concept by presenting it is a *problem,* as something to be investigated which arises from a sense of inquiry and

curiosity. What happens in this problem-situation? What other situations do we know which are similar? What other situations do we know which are different? What information do we have about this problem-situation? How can different parts of this information be put together to get "new" (though already implicit) information? *Why* is this problem interesting? What *purpose* is there behind this problem? If we could find out more about this problem, what use would the new ideas be? What do the concepts behind the problem, and what does the problem itself mean? Do we need any new *concepts* to handle the problem?

We need to present mathematics as a purposeful, meaningful subject, in which things which are not known need to be found out.

We need to test our understanding of isolated ideas by deliberately linking pairs and groups of otherwise related ideas. In this way we not only multiply-*embody* a concept but also multiply-*apply* concepts. From this we develop higher level concepts and *higher*-level skills in the cognitive taxonomy of comprehension.

We need a new *attitude* to how and why we do mathematics, an escape from deductive formulas to heuristic inquiry.

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Part Two : SOME SUGGESTIONS FOR RESEARCH.

(i) The Double PNG Education System and Possible Research Options.

In any plan for educational research in Papua New Guinea, it is sensible to be aware of the two different education systems currently in operation, and to be prepared to exploit these systems for mutual comparison. The first of these two systems is the *National* schools, dealing with locally developed and monitored curricula, and catering almost exclusively for PNG citizen students. The second is the International school system, using expatriate curricula, and catering predominantly for expatriate students.

The differences in curricula between these two systems make it possible to compare the effectiveness of curricula between the two systems. Where one of the systems teachers one piece of curriculum which the other does not teach, we can use the latter system as a *control group* for the first. Where the two systems teach basically the same curriculum, though perhaps in different ways, we can compare the two systems as alternative *experimental groups*.

Because up to one quarter of the *international* school population consists of *national* students, it is possible to carry out fairly direct comparisons of PNG students in *both* systems, though we must keep in mind other variables such as parents' socio-economic background.

Parents with children in international schools must be able to pay the *very* high school fees. These parents are also likely, if only because they are able to earn high incomes, to have had a good education themselves, and so can provide better support at home for the school's efforts. Similarly international school teachers usually have better or longer training than national school teachers. Also international schools have far more extensive resources and more money available than have national schools. These differences are important. But they must *not* be used as excuses for *not* making comparisons. The research information which *could* be obtained is *too valuable* to be simply ignored or thrown away.

The national students in international schools provide a very clear indication of what *can* be achieved by PNG citizens under very favourable circumstances. This is valuable information to have and it should be used.

Bob Roberts' paper "Community School Mathematics in Papua New Guinea: The International Connection" is just such a comparitive pilot study, looking at a very specific area of school and mathematics learning. By noting the *overlap* which exists in mathematics curricula between national and international schools, Bob was able to test *teachers* and *children* in a range of national and international schools at grade 2 to 6 level, on a general mathematics *achievement* test.

He was able to show that there was reasonable parity in achievement between the two education systems in grades 2 and 3. This

suggests how effective the junior primary mathematics instruction is in national schools. However after grade 3 there were significant advances by international school students.

Grade 4 is about the time when children in international schools have mastered the skill of reading, have become independent readers, and are able then to do an enormous amount of *general independent* learning about the world, through wide reading, having fairly free access to a wide range of reading materials, both fiction and non-fiction. By contrast, national students' access to reading material is extremely limited, both at school and home. (See also Jones' 1981 pilot study on reading ability in national and international schools in Clarkson 1981).

What might happen if a *Book Flood* could occur in PNG schools, as was tried with great success in Fiji, remains to be seen. In Fiji, $6000 of children's books were provided to grades 4 and 5 in eight schools and used for "wide reading", that is silent reading for pleasure. After only 8 months there were significant increases in many language and other curriculum areas, with *lasting* results, by comparison with control group schools which did not receive the books. (See W.B. Elley and F. Mangubhai *The Impact of a Book Flood in Fiji Schools* The Institute of Education, University of the South Pacific, 1981).

Bob Roberts made further comparisons between national and international teachers (the former knowing *significantly less* mathematics than the latter), and between PNG children in national international schools (the latter achieved significantly better than the former, and were in fact *keeping up* with their expatriate class peers!). But this is, as he admits, only an early,

pilot study.

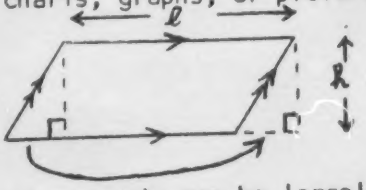
There remains a very large scope for further research using the national and international schools as experimental comparisons and control groups. The full range of primary and secondary schooling can be examined. So too can such other areas of mathematics learning as *attitude to mathematics, learning ability, learning processes, spatial ability, verbal ability, problem-solving strategies, text readability, concept development,* and so on.

As a *general guide line* it is possible to urge that, as far as possible, whenever any testing of the national schools is proposed, that this testing should also be carried out in the international schools. For example, this would be a useful addition to the research proposals in Godfrey Yerua's paper "Development and Implementation of (Mathematics) Textbooks in PNG "(as would the decision to test *across* grade levels).

(ii) The Multi-Modal Nature of Mathematics.

It is common to hear mathematics education researchers speaking of the learning of mathematics as being similar to the learning of a *foreign* or *second language*. Certainly, looking at the plethora of specialised terms that are involved in mathematics, such as "set, product, order, power, point, rational, factor," and "function", there is some truth in this. But, even in second language learning there is more involved than just memorising a specialised dictionary. Behind the words are concepts, which are themselves not always exclusively *verbal*.

We must remember that mathematics concepts can be thought of as either *symbolic*, that is, expressible in concise and formal mathematical symbols (e.g. $ABC = 125^{o}\ 13'$, $C^2 = a^2 + b^2$, $\overline{AB} \perp \overline{MN}$), or *verbal*, or expressible in words (e.g. the set of positive whole numbers smaller than one hundred which are exactly divisible by two), or as *visual* or *spatial*, that is, expressible in diagrams, charts, graphs, or pictures e.g.



Of course most mathematical concepts can be largely expressed in either symbolic, or verbal or spatial terms. (*Exercise* : Are there any mathematical concepts which cannot be represented in all three ways ?) As mathematics *teachers* we should work hard to show each concept we teach in symbols, *and* words, *and* pictures, because different children learn more effectively in *one* of these modes rather than in the other two modes. We need to optimise the efficiency of their learning by matching the mode of instruction to their favoured mode of learning. Also we need to increase the flexibility of their thinking by teaching them how to use their unpreferred modes more effectively. Indeed, we will see shortly how we can *exploit* the multi-modal nature of mathematics to teach it more meaningfully.

As mathematics education *researchers* we should be careful that our research does not unwittingly overlook one or more of the three modes. We may, for example,want to concentrate for research purposes on *spatial* abilities, separately from the other two. But we must keep in mind that spatial problems can

also be solved by non-spatial means. And if we are interested in solving *general* mathematical problem solving ability, we must include all three modes as possible strategies for solving problems.

Part Three : SOME SUGGESTIONS FOR TEACHING.

(1) Conceptual Cross-fertilization : A New Way To Use Old Materials.

What can we do if we want to teach mathematics in a more meaningful way ? if we want to connect separate recipes ? if we want to provide multiple embodiments for mathematical concepts ?

One way to do this is to deliberately *cross-fertilize* the otherwise separate recipes that we find in text books.

For example, suppose we take the list of concepts we find in the Contents pages of *Mathematics Our Way* 7A and 7B (1981), the new PNG high school text book : Patterns, Counting Systems, Place Value, Whole Number Arithmetic, Fractions, Decimals, Percentages, Length, Weight, Time, Area, Capacity, Angles, Coordinates and Statistics. If we write these in a grid horizontally, and vertically, we can then find that a greatly expanded range of problem types are possible. See Table I .

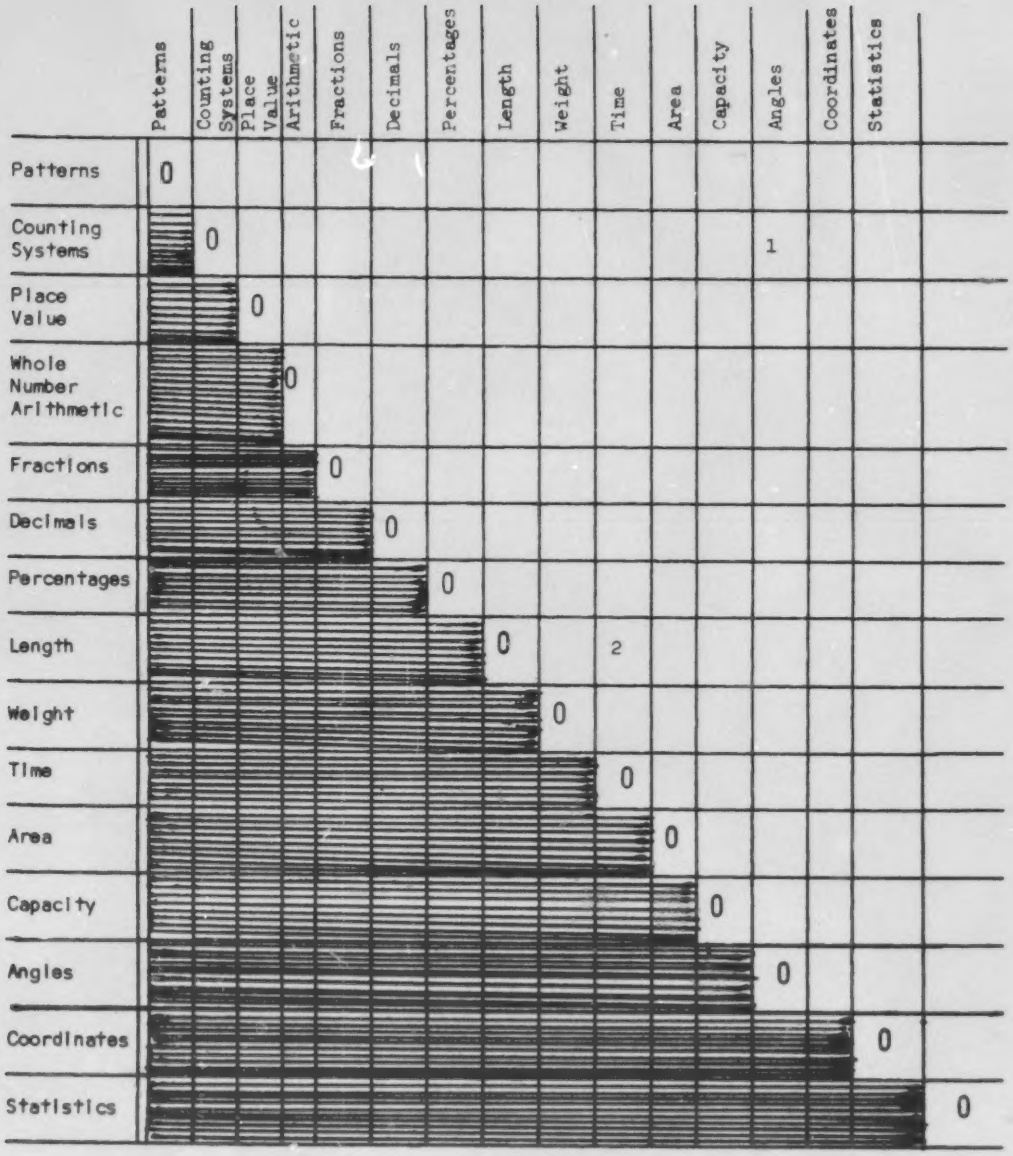


TABLE I : CONCEPT GRID FOR *MATHEMATICS OUR WAY* 7A,B

In Table I we can ignore the shaded part of the grid which simply repeats the unshaded part. Also we may observe that the bulk of the problems, exercises and teaching in *Mathematics Our Way* 7A and 7B come from the grid cells in the main diagonal which are marked with a 0 . These are the grid cells which have the same concept description for horizontal and vertical concept labels e.g. Patterns - Patterns, Place Value - Place Value. This is typical of most text books. It is a simple reflection of the usual way that concepts are presented, chapter by chapter, more or less in isolation from other concepts.

Inevitably there is *some* unavoidable linking of concepts in most textbooks. For example there is some amount of simple counting, or Whole Number Arithmetic, in the work on Patterns. Similarly it is almost impossible (and indeed silly) to teach Decimals in isolation from Fractions, or to teach Percentages separately from Decimals, and Fractions. Nonetheless this linking is usually, regretably, kept to a minimum.

Indeed, when one Concept is being taught as the major concept, other *necessary* concepts are only involved at a very low level of complexity. For example, in teaching Percentage, only very simple decimals and fractions are used to explain how percentages work. This is understandable in the *very* earliest stages of learning a new concept. But if it means that this is *all* a student is expected to learn in one year, then he will learn only a very small part of the whole concept. He will think that the concept is *simpler* than it really is, and may have considerable difficulty learning the real complexities of the concept

when (and if) he encounters them later. He may even be able to "learn" the over-simplified concepts in a wrong way which happens to work for these simple examples but which will not work for more complex ones.

For example a student may learn a rote method or recipe for handling percentages which is based on simple small percentages which as 5%, 10%, 25%, 80% and fail later to be able to understand or use other percentages such as 0.1%, 12.5%, 150%, $33\frac{1}{3}$ % and so on. If students are to be able to learn mathematics with proper understanding they must be given a chance to do so, by being deliberately exposed to the complexities of what they are learning.

It is up to the teacher, who should know the full complexities (or most of them) of the topics he is going to teach, to choose a sensible balance between wholly confusing his students by giving them *too much* of the complexity of a topic at one time, and misleading them by giving them *too little* of the complexity at a time.

Bruner's famous *spiral curriculum* is an excellent teaching strategy, particularly for remedial and revision purposes, matching the developing complexity of concepts in a subject to the developing cognitive ability of the students to handle the successive complexities. But it should *not* be used as an excuse for giving only a tiny glimpse of a concept at one time, and another tiny glimpse later on, and a further tiny glimpse even later, and so on, hoping that the learner will somehow be

able to piece the fragments together. The sensible *survival* learning strategy to use, in this situation, is to learn *one* fragment at a time, however that might be possible, so that the conceptual linking *intended* by the spiralling accretion of tiny bits may all too easily simply not take place.

To return to Table I . We can use *each* of the non-major diagonal cells to stimulate our thinking, as teachers, about possible new problems. Each cell combines two different concepts, challenging our students' understanding of separate recipes, and so making it possible to develop conceptual links between the concepts that will make any one concept more meaningful.

For example, with the cell for Counting Systems and Angles, the cell labelled 1 , this may suggest using degrees, minutes and seconds as an example of a counting system which combines base 10 and base 60 . The same sort of idea holds for Time and Counting Systems, with a 365-12-7-24-60-60 "base" system for years, months, weeks, days, hours, minutes and seconds.

Another example could be Time combined with Length, in the cell labelled 2. Here we have the possibility of problems based on speed and growth. The same idea holds for Time and Weight, Area, Capacity, Angles.

Of course graphs, using Coordinates, can be used for *all* the other concepts, as can *Statistics* .

Such a grid makes it clear that, by and large, our approach to concept presentation has been limited so far to a fairly linear

sequence of isolated concepts. Not only does this *not* have to be the case, it would actually improve learning if we began to cross-fertilize our thinking and our students' learning.

(II) Cross-Modal Conceptualization and a Researcher's Approach to Finding and Solving Problems.

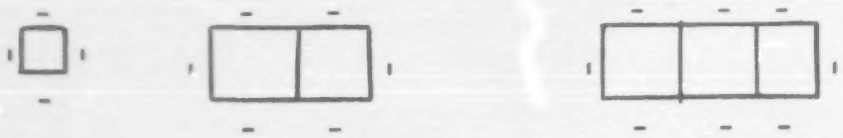
Another way to stimulate meaningfulness in mathematics learning is to ask the *cross-modal* question: *How can I express this differently, in symbols, words or pictures ?* This has the advantage that, as we are learning a new concept in one mode (symbol, words or pictures), we can also understand it in another mode, or relate what we are learning to concepts we already know in another mode. We see this happening very fruitfully when algebra meets geometry, symbols meet pictures, in *coordinate geometry*, and we investigate circles and lines and so on as functions and equations.

And yet another way is to ask the kinds of *curiosity - questions* used by creative researchers: *Is this always true ? Are there counter examples ? How could I change this, and what would happen if I did ? Are there other situations like this ?*

Let me give some examples, drawing on some of the problems already considered in other parts of this conference.

Bob Hyde's Table and Chair Problem

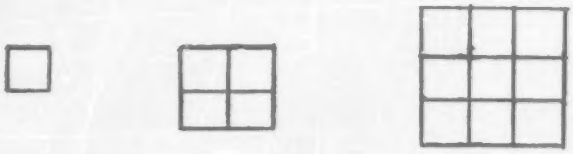
Given one square table we can seat four people, and two square tables seat six people, and three square tables seat eight people.



Bob's problem was, how many chairs can we fit at 41 or 90 or *N* tables ?

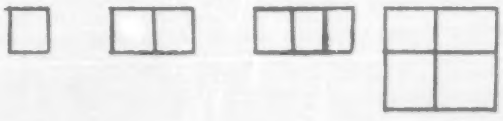
But we can generalise the problem in many ways. How could we change this ?

We could ask for tables to be fitted together in successively larger squares :



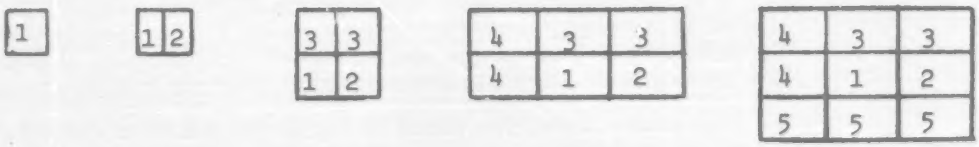
etc .

We could ask the for tables to be fitted together in rectangles or squares of successively larger area :

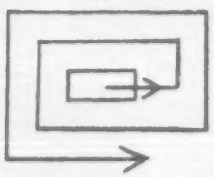


1 2 3 4

We could ask for extra tables to be added in successive *spirals* , so that each time a whole side was added to make a complete rectangle or square :



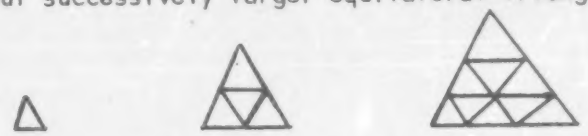
etc.



We could ask for the tables to be equilateral triangles :

etc.

What about successively larger equilateral triangles ?



etc .

Are there other situations like this ?

Clearly this is equivalent to a Perimeter - Area problem, where the perimeter is being measured, in unit lengths, by the number of chairs. We can ask questions such as: Which table arrangement gives the most amount of chairs for the fewest tables ? (I.e. maximise perimeter and minimise area) which way of arranging gives the fewest chairs for the greatest number of tables ?

Of course such a 2-dimensional problem readily generalises to the 3-dimensional problem of painting exposed faces of solid shapes made out of more and more unit cubes. There are the questions of maximising volume and minimising surface area, and vice versa.

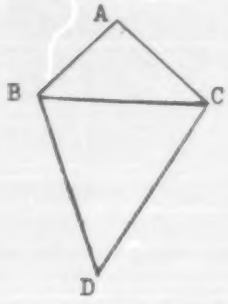
How can these situations be expressed differently ? We can ask students to work out symbolic formulae, or word rules for some of these situations. We can ask students to *graph* the number of tables *(x)* against the number of chairs *(y)* . Of course we can ask students to construct tables of values, for tables and chairs.

Lilia Dube's Triangle Problem.

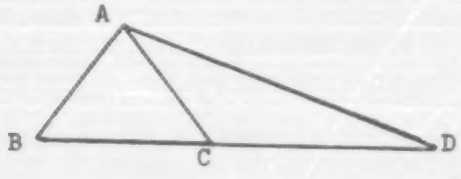
Given that triangle ABC is equilateral and triangle DBC is isosceles and angle D is half of angle A, Lilia asked her students to *prove* the length of AD is the same as the length of BC .

But this gives the *target* solution. We could simply pose the problem of deriving as much new information as possible.

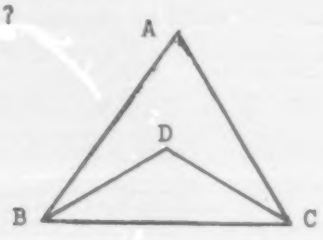
What if D is on the other side of BC ?



What if D is on BC extended (and triangle ABD *not* isosceles): what can we say about the length of CD, AD ; where angle D is half of angle BAC ?



What if isosceles triangle BDC is inside triangle ABC and angle BDC is *twice* angle BAC ?



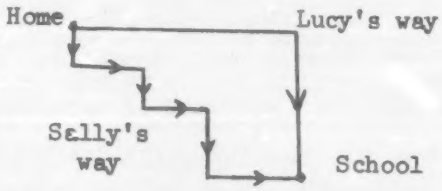
What if we generalise further and allow D to lie on the perpendicular bisector through BC, and we allow D to move along that bisector : what happens to angle BDC and the length of DC ?

Can we answer this by scale diagrams, or by simple trigonometry and Pythagoras theorem ?

Can we make a graph of the length DC (x) with the angle BDC (y) ? A polar coordinate system would be the *natural* system for such a graph, but would it show any asymptotic behaviour as well as a Cartesian Coordinate System ? Can we write a formula for thse graphs ?

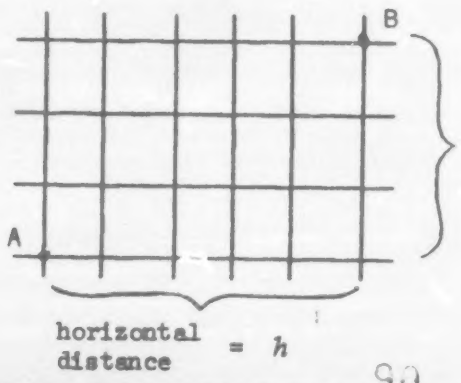
Harold Fredrick's Short-Cut-to-School-Problem.

Sally thinks she has a short cut to school because she is going along the short side of a "triangle" whereas Lucy goes along the two long sides.



This problem generalises to the problem of *shortest paths* in a square-grid city. E.F. Krause's delightful book *Taxi-cab Geometry* (Addison-Wesley, 1975) discusses this generalisation, and gives excellent practice in coordinate reading and plotting, and acute insight into the usual Euclidean way of measuring distance.

In a grid city a taxi can only travel along *roads* , so it cannot go diagonally. This means that the *taxi distance* from points A to point B is the sum of the horizontal distance separating them, and of the vertical distance separating them.



There are shocks ahead when we try to plot the set of points whose *taxi distance* from A is 6 units - a taxi "circle" of radius 6 .

What is the taxi-distance equivalent of *pi* , π ?

What is the taxi-distance equivalent of an ellipse ?

Given two fixed points, what is the set of points whose taxi-distance from one point plus the taxi distance from the other point is equal to the taxi-distance between the two fixed points ?

What is the taxi-distance equivalent of the locus of points which are equi-distant from two given fixed points ?

There is much pleasure and learning, and challenge to existing geometric concepts to be had from an investigation of taxi-cab geometry.

Of course we can then ask similar questions about a *triangular* grid city.

CONCLUSION

This paper has outlined the case for new approaches to mathematics teaching and learning. It has suggested some guidelines for research. And it has proposed several methods for enhancing the meaningfulness of what is being learned, and for beginning to respond creatively to materials which are already available in order to provide more effective stimulation to students who are coping at the moment with large amounts of limited recipe learning.

Can we build on this ?

A PRELIMINARY STUDY OF THE MATHEMATICAL GENERALIZATION PROCESS IN LATE-SECONDARY PAPUA NEW GUINEAN STUDENTS

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ABSTRACT

Mathematical generalization is a thinking process fundamental to the achievement of mathematical competence. On the basis of an earlier pilot study the author proposes the hypothesis that the development of the ability to generalize mathematical material proceeds in a hierarchical stepwise manner. Tests of mathematical generalization ability have been devised and administered to a large sample of late secondary Papua New Guinean students with a view to testing the above hypothesis. The data collected to date provides evidence in support of the hypothesis. The data also indicates that the mathematical generalization process in late secondary Papua New Guinean students is strongly content dependent.

INTRODUCTION

The arguments for and against process-oriented curricula have been well aired over the past two decades. That mathematics curricula should seek to promote those thinking processes and strategies which reflect the knowledge-getting process and not solely be concerned with the transmission of known mathematics is now a widely held view among mathematics educators. Bruner (1960), an early exponent of the process-oriented curricula summarizes the arguments in its favour as follows:

> ".....the curriculum of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to that subject. Teaching specific topics or skills without making clear their context in the broader fundamental structure of a field of knowledge is uneconomical in several deep senses. In the first place, such teaching makes it exceedingly difficult for the student to generalize from what he has learned to what he will encounter later. In the second place learning that has fallen short of a grasp of general principles has little reward in terms of intellectual excitement. The best way to create interest in a subject is to render it worth knowing, which means to make the knowledge gained usable in one's thinking beyond the the situation in which learning has occurred. Third, knowledge one has acquired without sufficient structure to tie it together is knowledge that is likely to be forgotten. An unconnected set of facts has a pitiable short half-life in memory. Organizing facts in terms of principles and ideas from which they may be inferred is the only way of reducing the quick rate of loss of human memory" (Bruner (1960) p. 31).

An interesting overview of the process-versus-content arguments as they relate to mathematics education in developing countries, in particular Papua New Guinea, has been provided by Philp (1973). On the basis of his research into classification and problem solving strategies employed by Papua New Guinea children, both schooled and unschooled, he suggests that in non-Western societies and particularly non-literate areas, the curriculum should be process-oriented and methods be heavily based on discovery learning. He points out that, while operationally

clear objectives may determine methods, the objectives themselves ought to be determined as much by the total learning situation, including the capabilities of the child, as by the demands of the subject matter. Thus he argues that, where possible, curricula and methods should be devised to take account of the preferred learning processes and strategies of each particular cultural group. This may be a feasible approach in most primary schools in Papua New Guinea, where in a particular school the majority of students may indeed form a culturally homogeneous group. In the secondary schools however this circumstance rarely occurs and it is necessary to adopt less culturally specific objectives and methods. In requiring Papua New Guinea students to form and use Western-type concepts it is essential that the curricula and methods employed embody not only the concepts but also the ways of thinking which lead to them.

Research into the teachability of the process aspects of mathematics, as reviewed by Jeffery (1978) and Bell (1979), indicates that mathematical strategies and thinking processes are teachable. Few of the research methods employed to date however could be regarded as realistic classroom teaching techniques. The work of Bell (1976 a) is a noteable exception in this regard. By broadening the curriculum to include process as well as content objectives and employing suitable methods he has demonstrated that average 11 year old English children can acquire and selectively employ generalization strategies. He describes his method as follows:

> ".....to consider the range of worthwhile mathematical activity which pupils at the stage in question can undertake, to distil from this the strategies required and then to design some teaching round each of the resulting strategies. One needs to give sufficient problems embodying each strategy to enable that strategy to become recognised and conscious, then gradually to introduce a range of situations where a selection has to be made from the strategies known. Continuing in this way the use of a set of relevant strategies becomes a flexible acquisition." (Bell (1979) p. 365).

Bell's study and others undertaken by Jeffery (1978) at the Shell Centre for Mathematics Education at Nottingham have demonstrated that significant improvements in students' mathematical processes can be realized in the classroom situation through appropriately designed methods.

The present study is concerned with the nature of the mathematical generalization process as observed in late secondary Papua New Guinea students. The immediate objectives of this investigation are discussed below. In the broader sense the study represents a first step toward the identification and characterization of those process aspects of mathematical activity which are considered to be essential to the development of mathematical competence. The availability of research information of this kind will provide a basis from which curriculum officers and material writers can produce curricula and materials which reflect the inherent importance of the process aspects of mathematics.

THE PILOT STUDY

The format and style of presentation of the generalization test items employed in the present study was largely influenced by the results of a pilot study undertaken in August 1981 involving 76 grade 10 students from a Provincial High School near Lae. After several weeks observing routine mathematics classes and helping students with their difficulties, it was possible to select appropriate material from which to devise test items consisting of five or six tasks relevant to the generalization process. A score of 1 or 0 was assigned to each task response corresponding to success or failure on that task. A total item score was obtained by summing the task scores for that item. An underlying hierarchy was suggested for one of the items in that the responses to the five subordinate tasks resulted in a predominance of cumulative scores of the type 10000, 11000, 11100, 11110 and 11111. In order to further explore the strictness of this hierarchy a similar sequence of subordinate tasks was adopted in the items used in the present study.

Another point of interest arising from the pilot study was the surprisingly small magnitudes of the inter-item correlation coefficients. In all cases the Kendall Tau rank coefficients were less than 0.2, indicating little relationship between scores obtained on each of the items. It should be noted that no attempt was made to adhere to a consistent format and style of presentation in these items and perhaps this contributed to the poor relationship observed between item scores. It appears likely however, that the extent of the generalization process was substantially affected by the content of the item, despite efforts taken to select content likely to be familiar to most students. The content dependence of the generalization process is further investigated in the present study.

THE PRESENT STUDY

The present study is an attempt to measure and characterize the mathematical generalization process as inferred from the responses of a large sample of Papua New Guinea National High School students to two items developed by the author. The items, "polygons" and "triangular numbers", henceforth referred to as item 1 and item 2 respectively, are included in the appendix to this paper. Although the mathematical content of these items is substantially different, both employ the same format and style of presentation, the sequence of tasks to be performed reflecting the model learning hierarchy suggested below. Thus at least one of the variables present in the pilot study has been removed.

The sample chosen for the study consists of about 400 grade 11 and grade 12 students currently studying mathematics at one of the National High Schools. At present data collection is incomplete. Of the total of 272 students for which some data are available, 75 are females and 58 are grade 12 students. Item 1 data are available for 265 of these students, item 2 data are available for 161 students and a complete set of

data (item 1 and item 2) is available for 154 students. Whilst this sample is large enough to make some statistically valid observations, the analysis presented in this report should be regarded as preliminary in nature. A more detailed analysis will be undertaken and reported when data for the complete sample becomes available.

In order to measure mathematical generalization a suitable instrument must be devised. The existence of such an instrument implies a knowledge of the sequence of subordinate intellectual skills comprising the learning hierarchy for mathematical generalization. The precise nature of the relationship between the psychological processes involved in the acquisition of mathematical concepts and the logical structure of the concepts themselves is still not clear. Hart (1981) warns that it is possible that sequencing of material by an expert who knows "the whole" is not necessarily the best way of matching the order of learning natural to a child. An alternative view is expressed by Suppes (1966):

> ".....the psychological stratification of mathematical concepts will seldom, if ever, do violence to the logical structure of these concepts; but it will markedly deviate from the mathematical analysis of the same concepts with respect to the amount of detail that must be considered"

The experience gained from the pilot study is consistent with this latter view and hence the sequence of subordinate skills comprising the suggested learning hierarchy has been chosen to reflect the logic sequence a mathematican might employ in arriving at the required generalization.

A MODEL LEARNING HIERARCHY FOR MATHEMATICAL GENERALIZATION

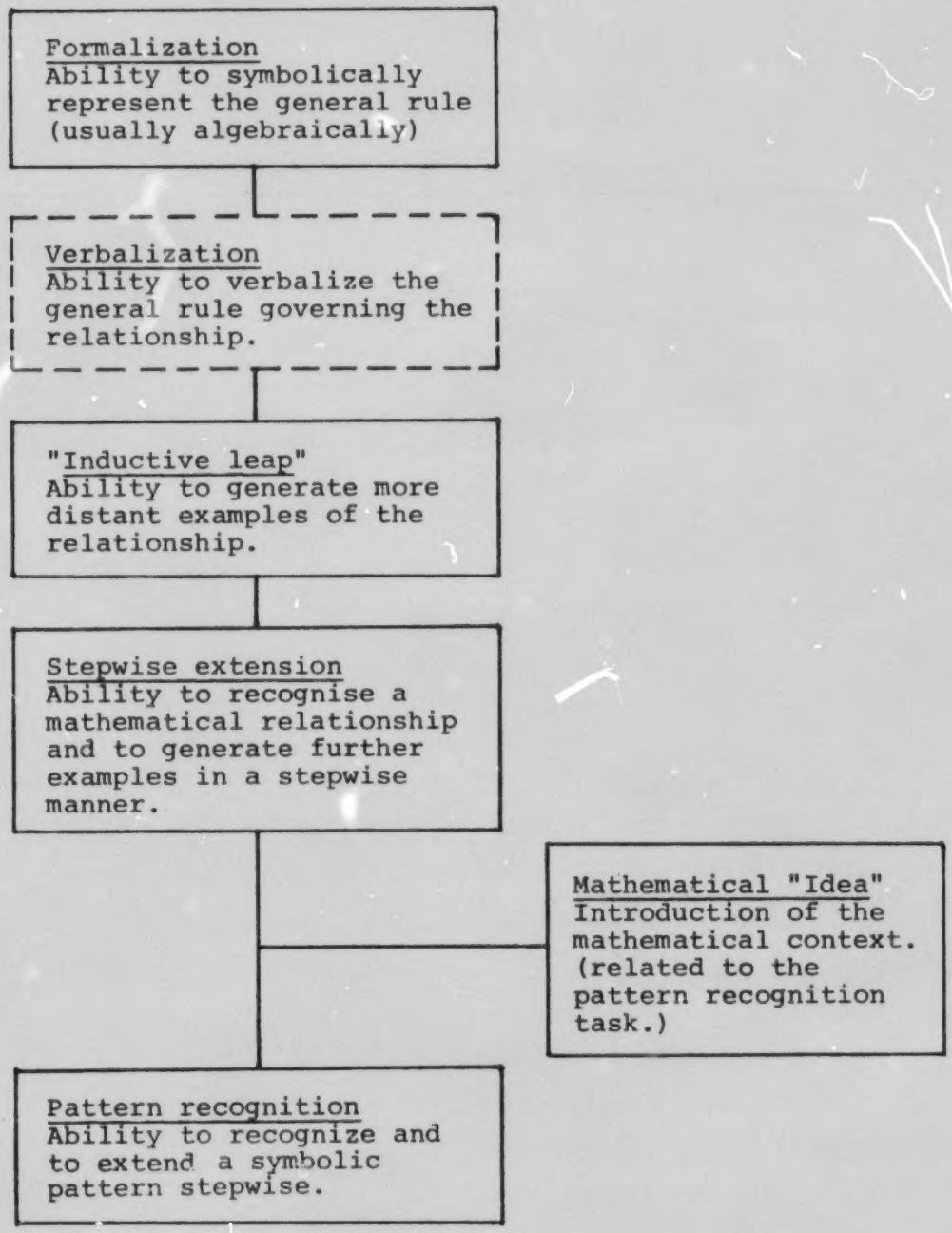
The model hierarchy proposed is illustrated in Fig. 1, it indicates the expected sequence of acquisition of five distinctly different intellectual skills thought to be fundamental to the process of mathematical generalization.

The intellectual skill associated with verbalization of the generalization is enclosed by a broken line to indicate that some doubt exists as to its proper place in the sequence. Wills (1970) states that if performance is foremost among our goals, verbalization may be delayed considerably in order to save time and possibly even the concept itself. He suggest that emphasis should be placed on the development of the symbolic level of expression and that premature attempts to verbalize might actually inhibit the learning process. In support of this view he cites the work of Schwartz (1948), who found that significantly diminished achievement on subsequent similar tasks occurred among subjects who were unable to give an accurate verbalization of their discovery. No significant decrease in performance was observed among those who verbalized accurately.

Bell (1976 b), in discussing mathematical generalization argues that verbalization is an important generalization ability in that it enables explicit expression of a relationship which is previously at the level of unconscious awareness. It is possible that verbalization of this sort might play a part in the development and consolidation of lower order generalization skills such as pattern recognition.

For nearly all of the students in the sample, English is a second or third language. One might expect that only a small proportion of those with an understanding of the general rule governing a mathematical relation would possess sufficient command of English to be able to express that understanding verbally. However a preliminary analysis of the student responses does not support this conjecture. For item 1, of the 265 students in the sample, 152 were successful to some extent on the "inductive leap" task. It might reasonably be assumed that only these students would be in possession of the the knowledge required to formulate the required generalization. Of these, 66 students or 45% provided a comprehensive and accurate verbalization of the generalization. Furthermore, of the 66 students able to

FIG. 1 MODEL LEARNING HIERARCHY FOR GENERALIZATION


Formalization
Ability to symbolically represent the general rule (usually algebraically)
Verbalization
Ability to verbalize the general rule governing the relationship.
"Inductive leap"
Ability to generate more distant examples of the relationship.
Stepwise extension
Ability to recognise a mathematical relationship and to generate further examples in a stepwise manner.
Mathematical "Idea"
Introduction of the mathematical context. (related to the pattern recognition task.)
Pattern recognition
Ability to recognize and to extend a symbolic pattern stepwise.

provide accurate verbalizations only 22 were able to represent the generalization symbolically. Of the total of 27 students able to represent the generalization symbolically only 2 showed no verbalization ability. These preliminary observations suggest that verbalization is a lower order skill than symbolic representation, supporting the proposed placement of verbalization in the model hierarchy.

The 'mathematical idea' to be generalized in each of the items is introduced after the symbolic pattern recognition task, as shown in Fig. 1. Its placement in the figure indicates its independence of the hierarchy. The earliest point at which it can be established that the idea has been assimilated to some extent is in the subsequent stepwise extension task. If the student's response to this task indicates that the idea has not been assimilated then the appropriate conclusion is that the student is unable to generalize in the context of that particular mathematical idea - not that he is unable to generalize mathematical material.

SCORING THE ITEM TASKS

Fig. 2 illustrates the most detailed scoring system employed in this study. Also shown is a brief verbal description of the extent of development of the generalization process skill corresponding to the score obtained on a particular task. The overall performance of an individual on either item can thus be summarized by a five digit code, eg. 12210, or by the corresponding list of verbal descriptions relating to the performance on each task, which for this example would be:

- able to extend symbolic pattern stepwise
- able to assimilate mathematical idea and confidently extend relationship stepwise
- confidently employs "inductive leap"
- verbalizes generalization in terms of specific cases
- unable to represent generalization symbolically

FIG. 2 DETAILED SCORING SYSTEM FOR GENERALIZATION ITEMS

Generalization process skill	Task Score	Extent of generalization process skill
Formalization	2	Correct symbolic statement of generalization
	1	Minor inaccuracies in symbolic representation
	0	Unable to represent generalization symbolically
Verbalization	3	Accurate verbalization in general terms
	2	Verbalization inaccurate or incomplete
	1	Verbalization in terms of specific cases
	0	No effective verbalization of generalization
"Inductive leap"	2	Confident "inductive leap" ability (2 distant cases)
	1	Limited "inductive leap" ability (1 distant case)
	0	No evidence of inductive leap
Stepwise extension	2	Confident extension of relation (2 steps)
	1	Limited ability to extend relationship (1 step)
	0	Unable to extend relationship
Pattern recognition	1	Able to extend symbolic pattern
	0	Unable to extend symbolic pattern.

METHOD A

In order to reduce the overall item performance categories to a manageable number, codes of the form X0000, XX000, XXX00, XXXX0 and XXXXX where X represents a non-zero digit, were categorized on the basis of the last non-zero digit occuring in the code. By this means the great majority of student responses to both items could be grouped into eleven categories. All responses falling into these categories were not necessarily hierarchic in the strictest sense. The small number of cases not falling into thes these categories were certainly non-hierarchic.

Table 1 shows that the distribution of responses over these categories is qualitatively similar for each item. The higher population of category 1 for item 2 indicates either that the next task i.e. extension of the mathematical relation is more difficult to master in the context of item 2 than in that of item 1 or that the mathematical idea involved in item 2 was more difficult to assimilate. For each item more than half the responses fall into categories 1, 5 and 8. These categories correspond to the highest developmental levels of the subordinate skills of stepwise relationship extension, inductive leap and verbalization respectively. These categories may be regarded as the more stable stages in the development of the generalization process in late secondary Papua New Guinea students.

METHOD B

An alternative scoring system further reduces the number of categories by collapsing two or more categories into one on the basis that responses indicating at least some degree of ability to employ a particular subordinate generalization skill might reasonably be grouped together. This approach yields six categories.

TABLE 1 ITEM RESPONSE DISTRIBUTION (SCORING METHOD A)

Category	Item 1 Freq.(%)	Item 2 Freq.(%)	Highest subordinate generalization skill achieved.	Task score for highest skill. (a)
0	9.1	3.7	}Pattern recognition	0
1	20.8	34.2		1
2	1.1	5.6	}Stepwise extension of relationship	1
3	11.7	7.5		2
4	5.7	0.6	}"Inductive leap"	1
5	20.4	19.9		2
6	1.1	2.5	}Verbalization	1
7	3.4	5.6		2
8	16.6	11.2		3
9	7.5	5.0	}Formalization	1
10	2.6	4.3		2
Sample number	265	161		

(a) For a verbal description of extent of highest order subordinate generalization skill corresponding to task score, refer to Fig. 2.

METHOD C: THE GUTTMAN SCALE SCORES

The Guttman scale analysis required to quantify the unidimensional and cumulative nature of the task responses necessitated yet another scoring system. The first stage of this analysis involves the determination of the cutting values for task scores in each of the items. Response scores greater than the cutting value are deemed to have 'passed' on that task, scores less than the cutting value are deemed to have 'failed'. For computation purposes 'pass' and 'fail' scores are recoded as 1 and 0 respectively. By summing the task scores for each item an overall item score in the range 0 to 5 is obtained.

With the exception of the verbalization tasks, a cutting value of 0.5 was assigned for all tasks on both items. For the verbalization task it was decided that verbalization in terms of specific cases and inaccurate or incomplete verbalization responses could not be regarded as sufficiently well developed levels of the verbalization skill to be ranked as a 'pass' score. Accordingly, a cutting value of 2.5 was assigned for this task.

Table 2 summarizes the item score distributions for each of the three scoring methods employed. The lower magnitude of the means observed for item 2 in each of the scoring methods is a reflection of the comparative difficulty of the second task in the context of this item.

TABLE 2 ITEM SCORE DISTRIBUTIONS

Scoring Method	A 11 Categories		B 6 categories		C 6 categories	
	Item 1	Item 2	Item 1	Item 2	Item 1	Item 2
Sample number	265	161	265	161	265	161
Score range	0-10	0-10	0-5	0-5	0-5	0-5
Mean	4.40	4.01	2.59	2.43	2.53	2.31
Std. deviation	3.07	3.07	1.49	1.44	1.44	1.35

GUTTMAN SCALE ANALYSIS

If the item tasks are in fact valid measures of the proposed subordinate generalization skills, then the degree to which the responses to the ordered tasks are unidimensional and cumulative, as determined by the Guttman scale analysis, quantitatively reflects the hierarchic nature of the proposed sequence. The degree to which the sequence is both unidimensional

and cumulative is assessed from the extent to which a 'pass' on any item task is associated with 'passes' on subordinate tasks. Each deviation from the expected hierarchic pattern is counted as an error. From the accumulated error score several standardized coefficients can be calculated which characterize the unidimensional and cumulative nature of the scale.

The results of the Guttman analysis for item 1 and item 2 are shown in Fig. 3 and Fig. 4 respectively. These figures also illustrate the method by which the various coefficients are calculated. In general a coefficient of reproducibility greater than 0.9 is considered to indicate a valid scale. The coefficient of scalability should be well above 0.6 if the scale is truly unidimensional and cumulative. For each item the calculated coefficients are safely above these values strongly supporting the hypothesis that the proposed sequence of subordinate generalization skills constitutes a valid hierarchy.

CONTENT DEPENDENCE OF THE GENERALIZATION PROCESS

The results of a paired two-tailed t-test on the 154-subsample, consisting of those students for whom both item 1 and item 2 scores were available, are shown in table 3. The null hypothesis, that the mean score obtained on each of the items is the same, is tested at the 5% significance level. The two-tail probability value of 0.03 indicates that a significant difference exists in the mean item scores for scoring method C. For scoring methods A and B the difference in mean scores is not significant at the 5% level. As a consequence of the marginal significance levels of the t-values observed, with respect to the chosen 5% level, the acceptance or rejection of the null hypothesis hinges on the choice of scoring system employed. Given that the scoring systems themselves are highly correlated, Kendall's Tau coefficients exceeding 0.94 in all cases, no definite conclusion can be drawn regarding the proposed hypothesis on the basis of the paired t-test results. Resorting to a more descriptive approach, comparison of the means and standard

FIG. 3 GUTTMAN ANALYSIS FOR ITEM 1

Task	5		4		3		2		1		
Score	0	1	0	1	0	1	0	1	0	1	
5	ERR 0	22	ERR 0	22	ERR 0	22	ERR 0	22	ERR 0	22	22
4	44	ERR 5	5	44	0	49	0	49	0	49	49
3	81	0	81	ERR 0	0	81	0	81	0	81	81
2	34	0	34	0	34	ERR 0	0	34	0	34	34
1	55	0	55	0	55	0	55	ERR 0	0	55	55
0	24	0	24	0	24	0	24	0	24	ERR 0	24
Totals	238	27	199	66	113	152	79	186	24	241	265
Errors	0	5	5	0	0	0	0	0	0	0	

Coefficient of Reproducibility $= 1 - \frac{10}{5 \times 265} = 0.9925$

Min. marginal reproducibility $= \frac{1016}{5 \times 265} = 0.7668$

Percent improvement $= 0.9925 - 0.7668 = 0.2257$

Coefficient of Scalability $= \frac{0.2257}{1 - 0.7668} = 0.9678$

FIG. 4 GUTTMAN ANALYSIS FOR ITEM 2

Task / Score	5		4		3		2		1		
	0	1	0	1	0	1	0	1	0	1	
5	ERR 0	11	ERR 0	11	ERR 0	11	ERR 0	11	ERR 0	11	11
4	18	ERR 3	3	18	0	21	0	21	0	21	21
3	43	0	43	ERR 0	0	43	0	43	0	43	43
2	23	1	24	0	22	ERR 2	2	22	1	23	24
1	56	0	56	0	56	0	55	ERR 1	1	55	56
0	6	0	6	0	6	0	6	0	6	ERR 0	6
Totals	146	15	132	29	84	77	63	98	8	153	161
Errors	0	4	3	0	0	2	2	1	2	0	

Coefficient of Reproducibility $= 1 - \frac{14}{5 \times 161} = 0.9826$

Min. marginal reproducibility $= \frac{613}{5 \times 161} = 0.7615$

Percent improvement $= 0.9826 - 0.7615 = 0.2211$

Coefficient of Scalability $= \frac{0.2211}{1 - 0.7615} = 0.9270$

TABLE 3 PAIRED T-TEST AND KENDALL'S TAU INTER-ITEM CORRELATION COEFFICIENTS (154-SAMPLE)

	Item 1	Item 2	Item 1	Item 2	Item 1	Item 2
Scoring method	A		B		C	
Mean	4.64	4.08	2.71	2.45	2.66	2.33
Std. dev. of mean	3.05	3.10	1.45	1.45	1.41	1.36
Std. dev. of (difference) mean	4.04		1.91		1.84	
t-value	1.70		1.65		2.23	
two-tail probability	0.09		0.10		0.03	
τ (Kendall's Tau)	0.10		0.11		0.08	
two-tail probability	0.06		0.05		0.11	

deviations of the scores obtained on the items in each of the scoring systems, suggests at least qualitative similarity of the score distributions.

The small magnitudes of the Kendall's Tau inter-item coefficients, also shown in table 3, indicate that there is little relationship between scores obtained by an individual on item 1 and item 2. In short, it appears that the overall item performance scores are strongly content dependent and that an individuals performance on item 1 cannot be taken as a reliable indicator of his likely performance on item 2.

In investigating the content dependence of the mathematical generalization process, ideally the two items should be of equal mathematical difficulty in order to reduce confounding effects due to this variable. The item response distributions shown in Table 1 suggests that the mathematical context of item 2

is more challenging than that of item 1. The results of the t-test however indicate that it is doubtful that the difference in mathematical difficulty would be sufficiently large to seriously prejudice the underlying assumption that the items are of similar difficulty.

The content dependence of the mathematical generalization process is likely to be intimately related to the individual's total mathematical experience. In any educational system it is difficult to characterize the educational background of students in any given grade. In the Papua New Guinea context it is virtually impossible. The sample employed in this study includes individuals from each of the twenty provinces of Papua New Guinea. Most obtained their primary education at community or village schools, of which 2000 or so are spread throughout the country, often so remote as to be accessible only by light aircraft. The distribution of educational materials to these schools is in itself a major problem. Hence, it is probable that the mathematical experiences of individuals comprising the sample will be widely different. Closer examination of the item task responses reveals a substantial number of individuals who demonstrated higher order generalization skills in the context of one item but who were unable even to assimilate the mathematical idea involved in the other item, as determined by their responses to the second task. In fact, for such individuals only one measure of their generalization ability is available. The removal of such obvious cases of content dependent performances from the sample enables the more meaningful comparison of generalization performances of individuals who were able to assimilate the mathematical idea in both items. Such a comparison provides a more appropriate measure of the content dependence of the generalization process itself.

In table 4 the results of a paired t-test together with calculated Kendall's Tau inter-item coefficients are presented for the 71 - subsample consisting only of those individuals who were able to assimilate the mathematical idea in both items. Of the original 154 sample, 63 cases were rejected on

TABLE 4 PAIRED T-TEST AND KENDALL'S TAU INTER-ITEM CORRELATION COEFFICIENTS (71-SAMPLE)

	Item 1	Item 2	Item 1	Item 2	Item 1	Item 2
Scoring method	A		B		C	
Mean	6.18	6.01	3.46	3.39	3.39	3.23
Std. dev. of mean	2.22	2.32	0.95	0.99	0.90	0.97
Std. dev. of (difference) mean	2.86		1.19		1.18	
t-value	0.50		0.50		1.20	
two-tail probability	0.63		0.62		0.23	
τ (Kendall's Tau)	0.15		0.21		0.15	
two-tail probability	0.05		0.02		0.07	

the basis that for these cases only one measure of generalization ability was available, 40 were able to generalize in the context of item 1 only and 23 in the context of item 2 only. A further 20 cases were rejected, being unable to generalize in either context. The results of the paired t-test on the 71 two-context-generalizers indicates no significant difference in the mean item scores for each of the scoring methods. The small magnitudes of the calculated Kendall Tau inter-item correlation coefficients for this sample, although slightly larger than those for the complete 154-sample, indicates that the generalization process itself, at least in secondary Papua New Guinea students, is strongly content dependent.

Adopting a Piagetian perspecitive on formal operational thought, one might expect the mathematical generalization process to be largely independent of content, assuming of course reasonable familarity with the concepts and symbolism forming the content.

Why then was the generalization process observed to be strongly content dependent for the subsample consisting of the 71 individuals who were able to demonstrate their ability to assimilate the mathematical content of both items? Wason and Johnson-Laird (1972) in their own work on reasoning processes and in reviewing other relevant studies, were equally surprised to find that those processes were dramatically affected by the actual content of the experimental material. In discussing the divergence between their results and Piaget's theory of formal operations, in the context of the processes associated with hypothesis testing, they suggest that experimental evidence reported by Wason and Shapiro (1971) and by Johnson-Laird, Legrenzi and Legrenzi (1972) supports the view that

> ".....formal operations are, in fact, only elicited by familar tasks, and not cognitive skills which can be applied to any problem whatsoever. In other words, they are really practical rules rather than formal operations" (Wason and John-Laird (1972) p. 190).

The concept of horizontal décalage, which assumes that there is not a one-to-one relation between task and developmental stage, can of course be invoked to to render such experimental observations compatable with Piaget's theory. Such a concept, however tends to make the theory untestable and could always be used to explain a discrepancy. A more speculative explanation, according to Wason and Johnson-Laird, is that unusual tasks, when presented in abstract terms, may induce temporary regression to earlier modes of cognitive functioning. Finally, they conclude

> "Content is crucial, and this suggests that any general theory of human reasoning must include an important semantic component". (Wason and Johnson-Laird (1972) p. 245).

Whatever the ultimate theoretical reasons for the content depence of reasoning processes, it seems clear from the above discussion, that cognitive functioning at the formal operational level is dependent on an underlying task-content interaction. The application of formal operations is most likely to be achieved when both the tasks to be performed and the content of the material are familiar. Hence the content dependence of the generalization process as observed in the 71-subsample is probably more attributable to lack of exposure to tasks of the type involved in generalizing mathematical material than to content related factors.

If the general applicability of mathematical processes like generalization are to be perceived and understood by the student, he must become familiar with the subordinate tasks involved by experiencing their application in a variety of contexts. In this way the process will eventually become divorced from the content and achieve the status of concept in itself.

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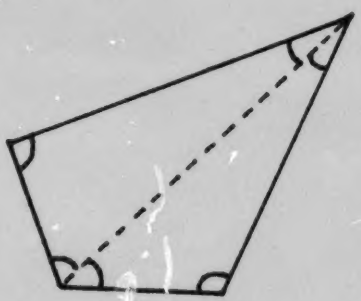
APPENDIX - ITEM 1

THE SUM OF THE ANGLES IN A POLYGON

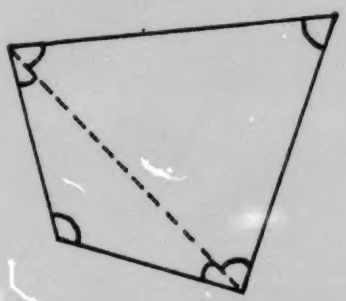
If you used a protractor to measure each of the angles in a triangle and then added these together you would get a total of 180^{o}. This result would be true for any triangle. So the sum of the angles in any triangle is 180^{o}.

Any four sided shape can be divided into two triangles.
Look at the examples below

(i)



(ii)



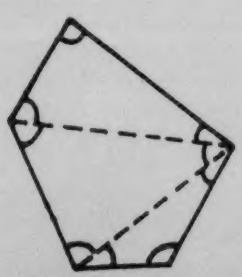
The sum of the angles in a four sided shape can then be found by adding up all the angles contained in both triangles. The sum of the angles in each separate triangle is 180^{o} and so the sum of the angles in any four sided figure is 2×180^{o} or 360^{o}.

This same idea can be used to find the sum of the angles in shapes with more than four sides. Such shapes are called polygons.

Follow through this idea for five and six sided polygons filling in the missing diagrams and blank spaces as you go.

Any <u>five</u> sided polygon can be divided into ..... triangles.
Here are some examples:

(i)

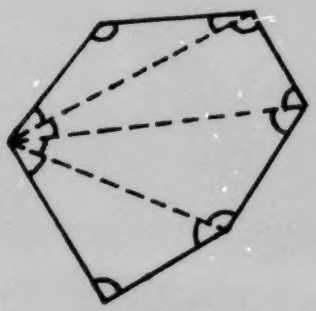


(ii)

For each separate triangle the sum of the angles is .....
So the sum of the angles in any five sided polygon is ..... $\times 180^{o}$
or .....^{o}

Any <u>six</u> sided polygon can be divided into ..... triangles.
Here are some examples:

(i)



(ii)

For each separate triangle the sum of the angles is .....
So the sum of the angles in any six sided polygon is ..... $\times 180^{o}$
or .....

Look carefully at the arithmetic expressions obtained for the sum of the angles in five and six sided polygons. Now without drawing diagrams write down the expressions for

(i) The sum of the angles in any seven sided polygon

(ii) The sum of the angles in any eight sided polygon

(iii) The sum of the angles in any 42-sided polygon

(iv) The sum of the angles in any 200-sided polygon

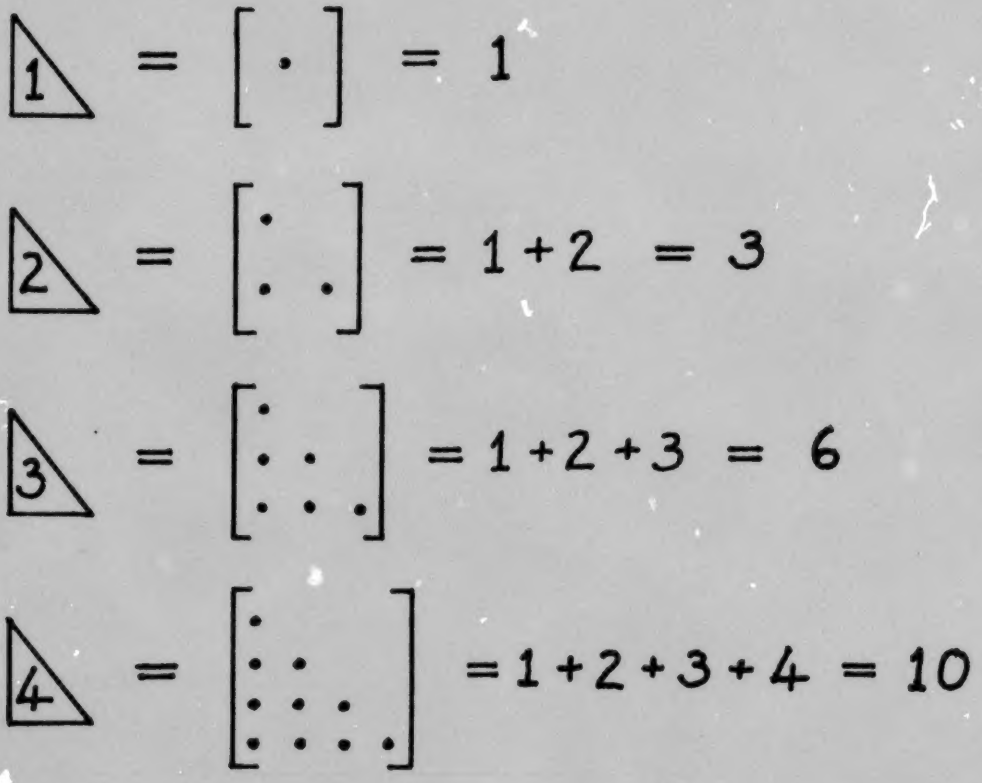
Imagine one of your friends is thinking about a polygon with a large number of sides. He knows how many sides it has got but you don't know. Describe carefully in words what you would tell him to do so that he could work out the sum of the angles in the polygon he is thinking about.

The letter "N" is often used to represent any whole number. Using this symbol write down the sum of the angles in a polygon with N sides.

ITEM 2

TRIANGULAR NUMBERS

Look at the following pattern of numbers in triangles, dots in boxes and ordinary numbers.

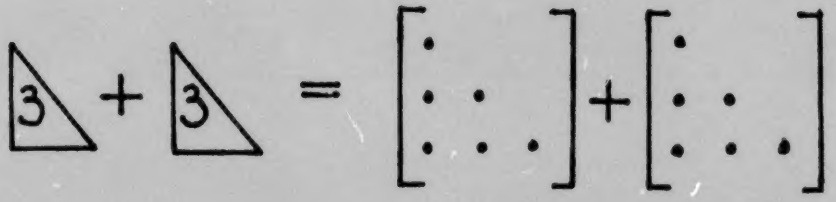


Now add another line to the pattern below

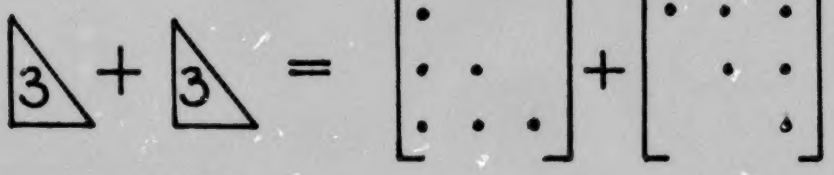
The triangles with numbers inside them are just symbols used to represent the number of dots inside the corresponding box. We call these numbers triangular numbers because the dots always form a triangle. By counting the number of dots in the box you can find the value of any triangular number. For large triangular numbers it would take a long time to draw and count all the dots. The following idea will help you find a quick way to calculate the value of any large triangular number.

Here is the idea

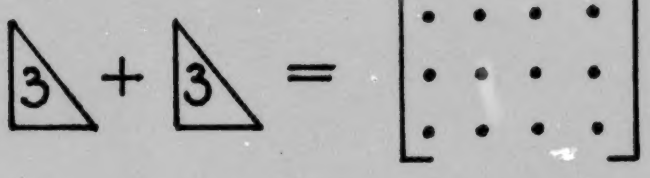
For we have



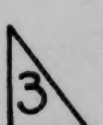
Since we are only interested in the number of dots we can rearrange them if we want to. Lets rearrange the dots in the second box to get

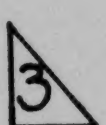


In order to count all the dots easily we can put them all together in one box

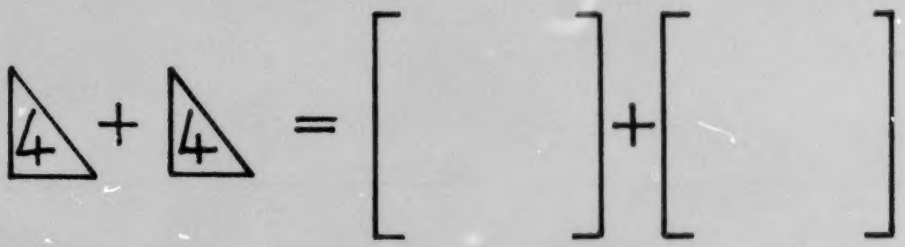


See how they form a neat rectangular pattern now. It is easy to see that the number of dots is equal to 3×4

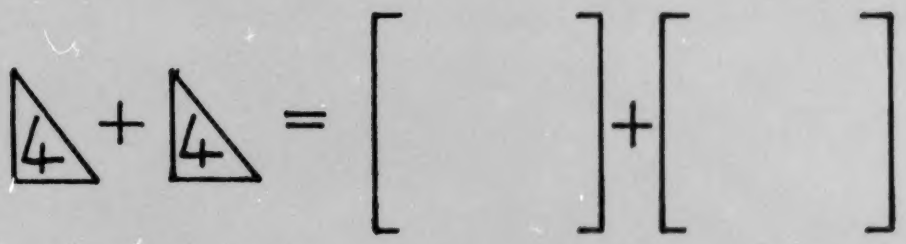
So $2 \times$  $= 3 \times 4$

therefore  $= \frac{3 \times 4}{2}$

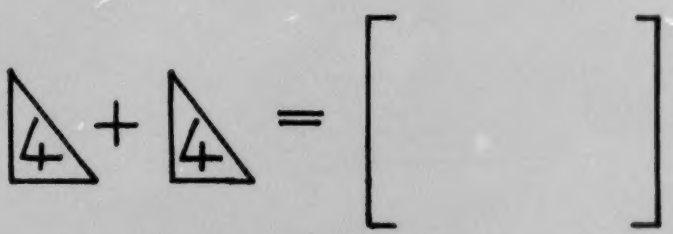
Follow through the same idea for filling in the missing dots and blank spaces as you go.



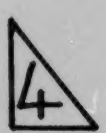
rearranging the dots in the second box we get

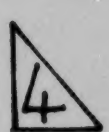


putting all the dots together in one box we get



We can see now that the number of dots is equal to .....X.....

So ..... X  = ...X.....

therefore  = $\frac{\text{.....X.....}}{\text{.....}}$

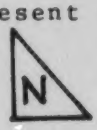
Look carefully at the arithmetic expressions obtained for

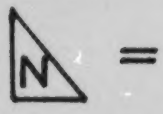
and

. Now <u>without</u> working through the idea from the beginning as you did for

, write down the arithmetic expressions for the following triangular numbers

Imagine one of your friends is thinking of a large number but you don't know what it is. Describe carefully in words what you would tell him to do so that he could work out the value of that triangular number.

The letter "N" is often used to represent any whole number. Using this symbol write down the value of 



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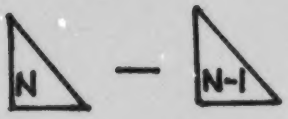
Work out the value of 

showing how you got your answer.

THE ROTATION AND DEVELOPMENT OF SURFACES: A STUDY OF A PIAGETIAN SPATIAL TASK WITH PAPUA NEW GUINEA SECONDARY SCHOOL PUPILS

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ABSTRACT

As part of a large investigation of spatial abilities of Papua New Guinea secondary school pupils, a series of geometrical solids were shown to 389 pupils in grades 7 to 10 in 5 provincial high schools and the pupils were required to draw the net or developed surface of each solid. This particular Piagetian spatial task has received scant attention from researchers and there is little cross-cultural evidence available. The results of this study were at variance with those obtained by Piaget's Geneva sample; in particular, the ability to develop surfaces occurred later with the Papua New Guinea sample than with the Genevan sample. Marked sex-differences were also apparent.

INTRODUCTION

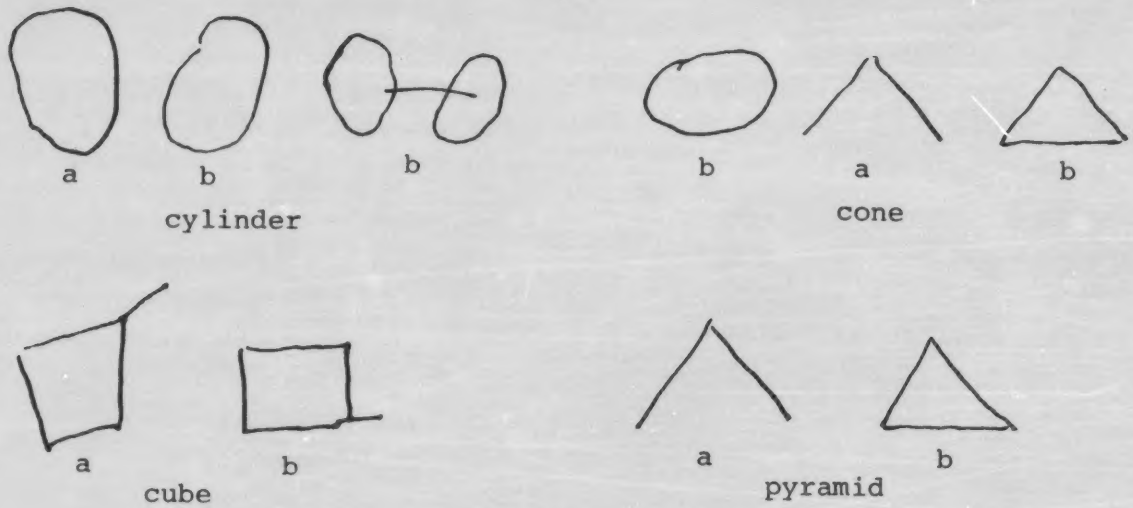
By comparison with studies of conservation, classificatory behaviour, and studies of proportional reasoning, the spatial and geometrical aspects of Piaget's work have received relatively little attention. A major work by Laurendeau and Pinard, The development of the concept of space in the child

(1970), followed the publication of Piaget and Inhelder's The child's conception of space (1956) and Piaget, Inhelder, and Szeminska's The child's conception of geometry (1960), but even this work investigated only a small proportion of the various tasks described in the two Piagetian works.

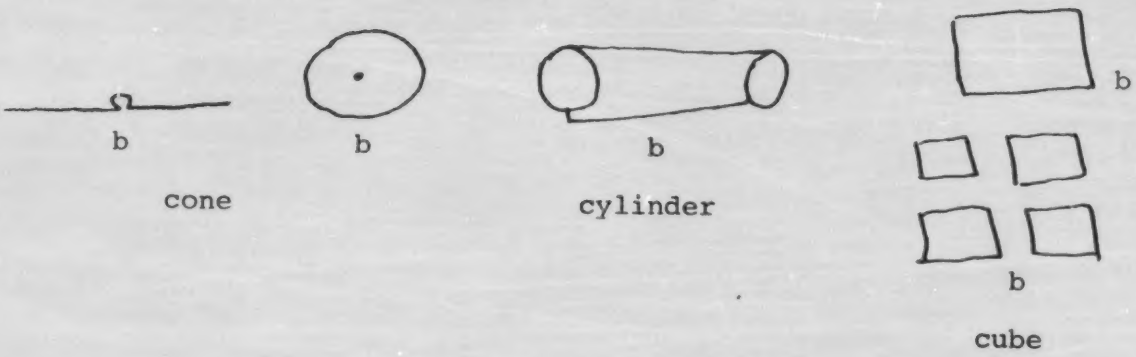
Various studies by Dodwell (1963) and Cowley and Murray (1962), for example, have investigated a small range of spatial tasks whilst other have investigated a particular task such as predicting the shape of a cross-section of a solid (Boe, 1966; Davis, 1973; Davies, 1976) or the ability to coordinate perspectives (Coie et al, 1973; Cox, 1975, 1977a, 1977b, 1977c, 1977d, 1978; Eiser, 1977; Elliot and Dayton, 1976; Fishbein et al, 1972). Also, by comparison with conservation studies, there has been scant attention paid in the cross-cultural literature to empirical studies of Piaget's spatial-geometrical ideas, exceptions being articles by Cowley and Murray (1962) and Omari (1972). The study to be reported here, an investigation of the development of surfaces of solids with Papua New Guinean school children, is therefore of interest in that it provides a cross-cultural perspective on a spatial-geometrical task which has received little attention outside Geneva.

THE PIAGETIAN VIEW OF THE TASK

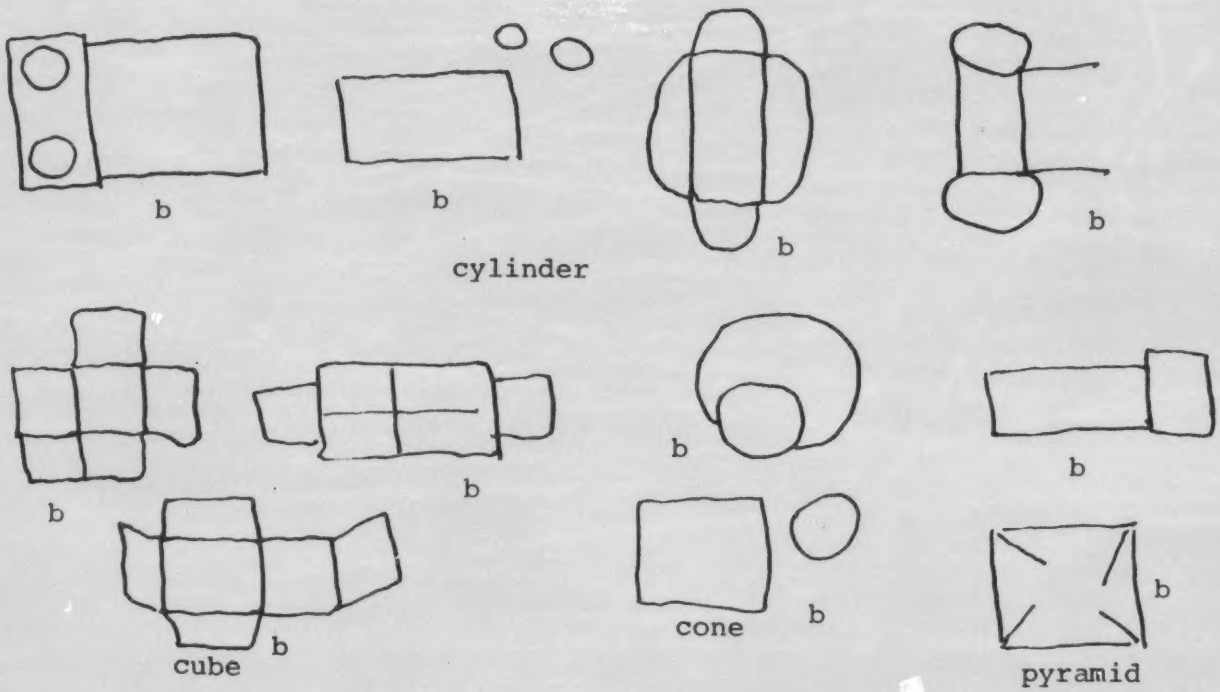
The task, which is described in detail in The child's conception of space (Piaget and Inhelder, 1956, pp. 271-302) involves presenting the subject with a three-dimensional model of a geometric solid and asking him to draw it as it would appear opened out flat; if he finds this too difficult, he is asked to describe how it would appear. After one or two trial solids, the subject is shown, in random order, a cylinder, a cube, a pyramid, and a cone. Within the context of Piaget's theory of the development of spatial cognition, the ability to successfully perform this task with the various solids is an outcome of the child's developing projective



Sub-stage 11A



Sub-stage 11B



Sub-stage 11A

FIG 1: Stages in 'rotation' and 'development', from Piaget and Inhelder (1956) pp. 274-275. (a = side elevation, b = developed surface)

REVIEW OF THE RELEVANT LITERATURE

It is only recently that studies specifically dealing with this task have appeared in the literature. Guay (1975) examined the relationship between the two Piagetian projective space operations: the coordination of viewpoints and the rotation and development of surfaces. Subsequently, Guay and McDaniel (1977) investigated the relationship between these, and other spatial skills, and performance on mathematical tasks with elementary school children. The only study which comes near to a straight replication of the Piagetian task is that by Faw (1977). The results of these studies will be discussed together with consideration of evidence relating to the effect on task performance of such experimental variables as age/grade, gender, and cultural factors.

Guay's doctoral study (1975) employed a design which had eight males and eight females at each of seven age levels from 7 to 13 years. Two instruments were developed in order to assess the ability to coordinate perspectives and the ability to rotate and develop surfaces. Guay did not find support for Piaget's view that the ability to coordinate perspectives is a necessary prerequisite for the development of the ability to rotate and develop surfaces; indeed he found limited support for the reverse relationship. Moreover, Guay also found that the order in which the tests were taken was important, taking the surface development test first enhanced performance on the coordination of perspectives task compared with subjects who took the latter test first. Guay was primarily concerned with investigating theoretical issues and not with investigating the details of task performance such as typical responses to particular solids. The solids which were used for both testing procedures were (i) right square pyramid, (2) tetrahedron, (3) right rectangular prism, (4) right square prism (truncated), (5) right triangular prism, (6) right rectangular prism (truncated), (7) right circular cylinder, (8) right circular cone. An item analysis of responses

to the multiple-choice rotation and development of surfaces test indicated that the least difficult solid was the right square pyramid and that the most difficult was the right circular cone; these results are the opposite of Piaget's findings.

The Guay and McDaniel study (1977) used the two instruments developed by Guay and 90 students from grades 2 through to 7 were tested on these as well as two further spatial tests, Serial Integration and Embedded Figures, and a test of mathematics achievement. The results indicated that a relationship existed between performance on all the spatial tests and mathematics achievement for subjects in grades 5 and 6.

Faw's doctoral study (1977) investigated the development of surfaces task with subjects from grades 6, 8, and 10, ranging in age from 12 to 16 years. Three methods of assessing the ability to do the task were used: the Piagetian approach requiring the subjects to draw their responses; presenting the subject with a solid and requiring him to select a net from a set of drawings; and a paper and pencil test, similar to those used in psychometric studies, which showed a series of nets and the subject was required to choose those which could be folded to form a cube. Faw used twelve solids in the first two methods of assessment and these included the cone, cylinder, the cube, and tetrahedron in order to compare her results with Piaget's. Faw also provided a brief training program for the children in grades 6 and 8. The results showed that the training program significantly improved performance on the test requiring a drawing response for children in both grades; the grade 6 children also showed a significant improvement in their performance on the pencil and paper test. Without training, the children in grade 6 scored more than 70% on the drawing response test for two items only: the cube and the square pyramid. The children in grade 8 mastered,

in addition, the triangular prism, a triangular solid, and the tetrahedron. The children in grade 10 added the frustum of a pyramid and a right triangular solid. Less than 30% of the drawings for the cylinder and the cone were correct for the sixth graders. Faw reported that, for the individual solids, the patterns of response were generally the same with respect to age, ability, and sex.

An unpublished study by Anderson (1980) reported some results of a rotation and development of surfaces task, carried out with 72 children from grades 1 to 6, and indicated the difficulty in attempting to define stage descriptions for drawing responses due to their wide variety.

Nelson and Kieren (1977) reported that, for a study carried out with children ranging from 3 to 8 years of age, most of the children successfully constructed a cube, tetrahedron, and a dodecahedron when given the nets of these solids but that a majority of the sample were unable to predict whether various nets would be formed to make a particular solid.

In discussing the differential responses of various groups to the surface development task we will consider first the influence of chronological age and grade level in school. Guay's results supported the view that the ability to perform the surface development task and to coordinate perspectives both improve with chronological age and grade level. Guay indicated, however, that most of the 12- and 13- year-old subjects were not able to deal completely adequately with either of the tasks. Faw reported that there were increases in scores, on all measures of the surface development task, with increases in age for almost all combinations of ability and sex. Guay and McDaniel's study also reports increases in mean scores on the surface development test proceeding from grade 2 through to grade 7. Results from other studies dealing with the development of the projective space operations also support the observation of improvement in performance

with increasing age and grade level. Laurendeau and Pinard (1970), for example, found improved performance on the coordination of perspectives task with increasing age but that even by 12 years of age many subjects had not mastered the task. Similarly, Towler (1969) found that for the same task the ability to perform adequately appears at about 10 or 11 years of ago. In investigating the ability to predict the cross-sections of solids, which is regarded by Piaget as developing at much the same age as the surface development task, Boe (1966, 1968) found that scores on a cross-section task correlated significantly with age and that even by age 17 years some subjects has difficulty predicting the sections of a cone. Davis (1969), however, found that with a sample of children from grades 6, 8, and 10 the sectioning task had been mastered by the age of thirteen or grade 8 and found no significant differences in performance between children in grades 8 and 10. Davis's study thus had a measure of agreement with Piaget's results; Boe, Guay, and Faw, however, all indicate that the ability to perform tasks related to the projective space operations tends to occur at an age later than the 10 or 11 years-of age indicated by Piaget. All studies agree, nevertheless, that performance improves with age and grade level.

Piaget has not generally reported data relating to sex differences in performance on his tasks. A number of studies based on Piaget's work,and on the projective space operational development,report data relevant to this question. Laurendeau and Pinard (1970) found that sex-differences in the performance of a coordination of perspectives task occurred with the oldest children, in favour of males, in their sample of 2 to 12 year-olds. Abravanel (1973) found that, for the same task, there were no significant sex-differences in performance for children between 5 and 9 years of age. Boe (1966) found, however, that for a cross-sections task carried out with secondary school children the males' performance was superior to that of the females. These results, then, support the generally

held view that sex-differences in performance on spatial tasks exist and usually begin to occur at the onset of adolescence. Guay and McDaniel (1977) obtained results which, however, found significant sex-differences in favour of males in the performance of elementary school children on a surface development task and coordination of perspectives task but not on two other spatial tasks which required lower level skills. Guay (1975) reported that for subjects from 7 to 12 years of age the males as a group performed significantly better than the females on the coordination of perspectives task and also that 8- and 9-year-old males performed significantly better than the corresponding females on the surface development task but for 10- to 12-year olds there were no significant sex-differences. Faw (1977) reported that for her grade 6, 8, and 10 subjects performance on drawing nets of a given solid resulted in males scoring higher than females at each grade level.
The general trend of all these results is such that if sex-differences in performance on projective spatial tasks occur at all then they are in favour of males, occasionally for elementary school children but more usually for older children.

There are a number of studies which report cultural differences in performance on a number of Piagetian spatial tasks. Cowley and Murray (1962) studied five Zulu and five white children in each of eight age levels from 5 to 12 years. The children were tested individually on a range of tasks covering Piaget's topological space operations (3 sub-tests), projective space operations (8 sub-tests), and euclidean space operations (7 sub-tests). The authors reported that while a similar sequence of development appeared to occur for both groups it was nevertheless evident that topological, projective, and euclidean spatial relationships emerged at a later age level in the Zulu children than they did for the white children. The authors do not appear to have investigated sex-differences in performance for either their white sample or their Zulu sample.

Two further studies by Mitchelmore (1976), carried out with students in Jamaica, are of interest here. The first required 40 males and 40 females aged 7 to 15 years to draw five wooden models: a cuboid, a cylinder, pyramid, cube, and cone. The results of this Solid Representation Test indicated that for four of the solids (the cone being the exception) it was possible to delineate five stages of representation and that subjects showed a fairly clear stage progression as both age and school grade increased. The stage descriptions showed certain similarities to those of Piaget for the rotation and development task. Notable sex-differences were apparent with the males' performance superior to that of the females up to about grade 7. Sex-differences in favour of males were also evident in the performance of Jamaican high school students on a Three-Dimensional Drawing Test which required the representation of parallel and perpendicular lines. Thus it would appear from the limited evidence available that in non-western cultures there are trends in performance on spatial and drawing tasks that are similar to those found in western cultures with respect to age and sex-differences.

One other group of studies which have a bearing on the rotation and development of surfaces task are those which appear in the psychometric literature and concern performance on standard pencil-and-paper spatial tests of the surface development type. Two such tests are the Surface Development Test, which is part of the Kit of reference tests for cognitive factors (French et al., 1963), and the Differential Aptitude Test: Space Relations (DAT-SR) (Bennett et al. 1974). A typical item from the first test shows a developed surface and a diagram of its solid; the subject is required to state where a specified edge on the surface appears when the surface is folded to make the solid. A typical item from the second test shows a diagram of a developed surface together with five choices for its solid, the subject being required to choose the correct one. The skills involved in both these tasks are similar, although not identical, to those required for the Piagetian task. One obvious difference is the use

of a two-dimensional representation of solids in both tests as opposed to an actual solid for the Piagetian task. Nevertheless the tasks do have obvious similarities and the data available from the two psychometric tests appears to be relevant. From the data quoted in the manuals of each test it appears that performance on both tests increases with age and that sex-differences in performance, in favour of males, are clearly apparent. Fennema and Sherman (1977, 1978) have used the DAT-SR in a number of investigations of mathematical ability and have typically found the DAT-SR scores, for both sexes, to be moderately correlated with mathematics achievement scores for subjects ranging from grades 6 to 12. Data from Bennett et al. (1974) provides additional evidence of this relationship.

In summary, then, there are a number of studies which are relevant to and enlarge our knowledge of how subjects respond to the Piagetian rotation and development of surfaces task. The available evidence does not agree entirely with the results obtained by Piaget in two main ways. First, there is disagreement as to the order in which subjects are successful in performing the task for particular solids. Second, there is disagreement regarding the ages at which subjects are able to perform the task successfully, the general trend being that the ability to predict the nets of a set of solids develops somewhat later than the age range given by Piaget. In addition, performance on the task shows differential responses for males and females, the former usually exhibiting a superior performance with the difference becoming more marked during adolescence. This result is consonant with those obtained from studies involving various psychometric surface development tests. With respect to cross-cultural studies there do not appear to be any published results available specifically on this particular Piagetian spatial task; studies of other related spatial tasks do indicate, however, that the development of the projective operations shows trends in performance with respect to age and sex similar to those found in the non-Piagetian studies

of the surface development task carried out with subjects from western societies. Prior to this study, no results were available for Papua New Guinean subjects on performance of the surface development task. If the development of spatial operations follows a pattern similar to the development of those operations required for various conservation tasks, then from Shea's (1978) results we might expect Papua New Guinea subjects to successfully perform the surface development task somewhat later than their western counterparts. Finally, Faw's (1977) results indicated that subjects improved their performance on surface development tests as a result of training. Churchill et al. (1942) found that subjects' performance on a psychometric surface development test also improved as a result of their undergoing a course in technical drawing. Thus we might expect that a relatively task-specific training program will have a beneficial effect on the performance of the surface development task, this view supporting Piaget's contention that performance on the task is enhanced by experience. The educational implications of this will be discussed subsequently.

DESCRIPTION OF THE STUDY

The data reported here derives from a study in which students from grades 7 to 10 in five high schools in Papua New Guinea were administered seven spatial tests. The details of the study including summary results are reported in Lean (1980). The five high schools were Kimbe in West New Britain Province, Rabaul and Malabunga in East New Britain Province, and Arawa and Hutjena in the North Solomons Province. The tests which were administered were:

(1) the Piagetian prediction of cross-sections of solids, a 25-item test requiring a drawn response;

(2) the Piagetian rotation and development of surfaces of solids, an 8-item test requiring a drawn response;

(3) a paper-and-pencil formboard test;

(4) a memory task;

(5) a task testing concepts of area and perimeter;

(6) a Piagetian locus task requiring a drawn response;

(7) Elithorn's Perceptual Maze.

The test which concerns us here is the development of surfaces test; the other tests will not be discussed further except in so far as they are relevant to the test under discussion.

In the administration of the development of surfaces test, eight cardboard models of solids were used. In order of presentation there were:

(1) cube, (2) cylinder, (3) triangular prism, (4) tetrahedron (5) square pyramid, (6) parallelepiped, (7) cone, and (8) pentagonal prism. There were also two additional models, a

'roof' and a 'tunnel', which were used as practice examples prior to doing the test itself. A total of 389 students did the test which was given to them in intact classrooms. Sufficient time was allowed for each subject to view each test model before drawing his response. The responses were subsequently marked independently by two assessors who assigned 0 for a wrong response, and 1 for a correct response; thus the maximum possible score for the surface development test was 8. Statistical analyses were subsequently carried out on test scores obtained in this way. An analysis of the drawn responses was also carried out, in particular to determine whether they could be assigned to definite stages according to the criteria given by Piaget (see fig. 1).

In considering the results it needs to be borne in mind that the testing methodology used, whilst attempting to follow the Piagetian approach as closely as possible, dealt not with individuals in a clinical testing situation but with groups with from 10 to 25 students in each one. Also, the groups were intact classrooms and did not comprise randomly allocated subjects. The number of grades tested in each school varied: at Malabunga, Rabaul, and Kimbe, grades 7 to 10 inclusive; at Arawa, grades 7 to 9 inclusive; and at Hutjena, two grades 9 and a grade 10.

RESULTS

The analysis of the results for the test of the rotation and development of surfaces of solids will be presented in four sections. The first will be a comparison of mean scores between sexes, between grades, between age groups, and between schools. The second is an item analysis of correct responses according to age and to grade levels. The third is an analysis of the drawn responses, showing in particular typical errors for each solid. Finally, a brief summary of the relationship between the surface development test and the other tests in the battery will be discussed.

RESULTS BY SEX, GRADE, AGE, AND SCHOOL

Grade	Males			Females		
	Mean	S.D.	N	Mean	S.D.	N
7	2.89	1.66	46	1.97	1.77	35
8	2.96	1.86	49	2.12	1.81	32
9	3.79	1.82	91	2.81	1.67	48
10	4.59	1.93	51	3.68	1.80	34

TABLE 1: Results of the surface development test showing means, standard deviations, and numbers of subjects for males and females in grades 7 and 10.

Using a pooled variance estimate and two-tailed t-tests for inequality of means we obtain the following results:

(1) At each grade level the means of the males and females results are significantly different, in all cases the two-tailed probability being less than 0.05; in each case the difference is in favour of the males.

(2) Over the entire grade range, both males and females show significant increases in performance.

(3) In considering the differences between the grades, each grade considered pairwise with its neighbour, the males' and females' results are dealt with separately. For the males, there is a significant difference between the means for grades 8 and 9 ($p<0.02$) and between the means for grades 9 and 10 ($p<0.02$). For the females, there is a significant difference between the means for grades 9 and 10 ($p<0.05$).

AGE	MEAN	S.D.	N
12	2.72	2.11	18
13	3.00	2.07	44
14	2.72	1.94	88
15	3.26	1.76	119
16	3.74	1.95	66
17	3.86	2.07	36
18	3.85	2.27	13

TABLE 2: Results of the surface development test showing means, standard deviations, and numbers of subjects for all subjects ranging from age 12 years to 19 years.

Using a pooled variance estimate and a two-tailed t-test for inequality of means and comparing a group pairwise with its neighbour, the only significant result occurs between the 14- and 15-year-olds. It should be noted that for all age groups the mean scores are less than half of the total possible score. Over the entire age range there is a significant increase in mean performance. The following table presents the males' and females' results at each age level separately to see whether they follow the same pattern. Referring to table 3 we find that
using a pooled variance estimate and two-tailed t-test for inequality of means we find that when means are compared pairwise each with its neighbour:

1. For males there are no significant differences,

2. For females there are no significant differences.

AGE	MALES MEAN	S.D.	N	FEMALES MEAN	S.D.	N
12	3.11	2.15	9	2.33	2.12	9
13	3.63	1.95	24	2.25	2.00	20
14	3.19	1.86	53	2.06	1.84	34
15	3.68	1.74	66	2.67	1.61	52
16	3.91	1.96	45	3.38	1.91	21
17	4.04	2.19	27	3.33	1.66	9
18	3.89	2.32	9	3.75	2.50	4

TABLE 3: Results of the surface development test showing means, standard deviations, and number of subjects for males and females ranging from age 12 years to 18 years.

When not comparing each mean with its neighbour we find that there is a significant difference ($p<.05$) in the means of the performance of males between the 12- and 16-year-olds. For the females there is a significant difference ($p<.05$) between the means for 12- and 18-year-olds. Thus, considering the males' and females' results separately, it is apparent that no dramatic changes occur between consecutive age groups but that gradual improvements in means occur over a four to six year age range.

In referring to Table 4 we can see that overall there are no major differences between the five schools, at each grade level, when we consider the females' results. One school, Kimbe, had a mean score at grade 7 level which was significantly lower than the results for the other schools, and it also had a mean score at grade 8 level which was significantly lower than the results for Schools 3 and 4.

SCHOOL GRADE	1 $\bar{X}$ S.D.	2 $\bar{X}$ S.D.	3 $\bar{X}$ S.D.	4 $\bar{X}$ S.D.	5 $\bar{X}$ S.D.
7	2.50 1.72 (10)	- -	2.60 1.90 (10)	3.33 2.31 (3)	0.67 0.65 (12)
8	1.75 1.71 (4)	- -	2.75 2.05 (12)	2.86 1.57 (7)	0.89 1.05 (9)
9	2.83 1.12 (12)	3.00 1.41 (9)	2.20 1.66 (15)	3.43 2.51 (7)	3.40 1.95 (5)
10	- -	3.25 1.60 (12)	3.92 1.73 (12)	2.80 2.17 (12)	5.00 1.73 (5)

TABLE 4: Results of the surface development test showing means for females only displayed according to school and grade.

The schools were assigned numbers as follows:

1. Arawa (North Solomons Province).
2. Hutjena (North Solomons Province).
3. Rabaul (East New Britain Province).
4. Malabunga (East New Britain Province).
5. Kimbe (West New Britain Province).

SCHOOL GRADE	1 $\bar{X}$ S.D.	2 $\bar{X}$ S.D.	3 $\bar{X}$ S.D.	4 $\bar{X}$ S.D.	5 $\bar{X}$ S.D.	
7	3.40 1.71 (10)	- -	2.87 1.30 (15)	4.00 1.58 (9)	1.67 1.44 (12)	5<1, 3,4
8	3.15 1.46 (13)	- -	4.00 1.92 (13)	3.63 1.85 (8)	1.53 1.30 (15)	5<1, 3,4
9	4.25 2.12 (8)	3.91 1.49 (47)	2.40 1.84 (10)	3.13 1.81 (8)	4.33 2.17 (18)	3<1, 2,5
10	- -	4.36 1.50 (11)	4.75 1.60 (12)	4.56 2.83 (9)	4.63 1.98 (19)	

TABLE 5: Results of the surface development test showing means for males only displayed according to school and grade.

The pattern for the males' results across school and grade levels is similar to that for the females in that the School 5 results for grades 7 and 8 are significantly lower than the means for the other schools. In addition, at grade 9, the mean for School 3 is significantly less than the means for Schools 2 and 5. All other results, across grades, are not significantly different.

ANALYSIS OF CORRECT RESPONSES BY GRADE AND AGE

For each of the Tables below the test solids have been assigned numbers as follows:

1. Cube
2. Cylinder
3. Triangular prism
4. Tetrahedron
5. Square pyramid
6. Parallelepiped
7. Cone
8. Pentagonal prism

GRADE	N	SOLID 1	2	3	4	5	6	7	8
7	82	49	20	37	39	74	4	11	12
8	83	42	18	35	53	78	4	8	13
9	140	57	36	55	51	91	11	26	15
10	85	71	20	67	68	93	22	22	49

TABLE 6: The percentage of correct responses for each solid at each grade level.

From Table 6 it can be seen that:

1. The easiest solid for all grade levels was the square pyramid.

2. For all grades, except 8, the cube shows the next best response. Also, except for grade 8, the performance on the triangular prism and the tetrahedron are almost the same.

3. Thereafter, the order of difficulty is: the cylinder, the pentagonal prism, the cone, and the parallelepiped.

4. The overall-performances for grades 7 and 8 are comparable.

5. A substantial increase in performance occurs for most solids, the tetrahedron and pentagonal prism expected, between grades 8 and 9.

6. By grade 10 the percentage performance on each of the cylinder, cone, parallelepiped, and pentagonal prism is below 50%.

It is interesting to compare these results with those of Faw (1977).

GRADE	N	SOLID 1	2	3	4	5	7
6	36	83	28	67	67	81	6
8	36	89	36	86	72	100	8
10	32	88	56	84	84	91	16

TABLE 7: Percentage of correct responses for six solids. The data is derived from Faw (1977) who studied a sample of 104 American children at grades 6, 8, 10. The results for males and females have been combined and so have the high, average, and low ability groups at each grade level.

Comparing Table 7 with Table 6 it can be noted that:

1. Both the PNG and American samples show a similar pattern of results. Both groups have results which indicate that the easier solids to deal with are the pyramid, cube, followed by the triangular prism and tetrahedron.

2. Both groups have considerably more difficulty with the cylinder and the cone, with more subjects being able, however, to do the former. The difficulty persists, for both solids, through to grade 10.

3. The results of both groups appear to conflict with those of Piaget, first, in respect of the order of difficulty of the solids (Piaget indicating that the cylinder and cone are managed prior to the cube and the square pyramid), and second in terms of age. We expect, from Piaget's results, to find that children have achieved mastery of the surface development task, at least with respect to the solids dealt with by Piaget, by age 12 years or so. Both the American and PNG results do not find that this is the case.

4. Comparing the American and PNG results by grade it is evident that the PNG subjects find the task generally more difficult than their American counterparts.

AGE	N	1	2	3	4	5	6	7	8
12	18	56	28	50	39	78	0	11	11
13	42	52	33	38	57	93	2	12	24
14	88	45	25	42	48	77	8	19	11
15	121	55	22	52	58	82	8	17	17
16	67	63	33	55	52	93	16	31	33
17	36	69	19	64	53	92	12	19	31
18	13	62	8	54	54	85	23	15	38

TABLE 8: Percentages of correct responses for each solid at each age level from 12 to 18 years.

From Table 8 it can be seen that:

1. The general tendency is for performance on the surface development task to improve with age.

2. The only solid which appears to be mastered by the 12- and 13-year-olds is the square pyramid.

3. With the harder solids there appears to be a levelling-out in performance (although there is a slight increase between the 15- and 16-year-olds' performances). The inference to be drawn may be that, without further experience in this type of activity, most subjects will not improve their performance.

ANALYSIS OF DRAWN RESPONSES

A typical feature of Piaget's approach to his data is the analysis and classification of responses to a particular task, in particular investigating the nature of erroneous responses. As in Piaget's case, the data gathered here came from investigating the responses of subjects at various age levels and not from studying a particular sample of subjects longitudinally. Care must therefore be exercised in drawing inferences from cross-sectional studies regarding how an individual's ability to perform a particular task may develop over time. Nevertheless it is possible to classify typical responses into classes and order the classes into a hierarchy of stages, although the criteria set up to assign a response to a particular class must, to some extent, be arbitrary. One such classification scheme will be presented below. Further issues to be addressed here are whether or not the general features of errors typically exibited by subjects in studies by Piaget and others are reproduced by the Papua New Guinea sample; also, whether there appear to be qualitative

differences in the responses of males and females; and, finally, to what extent a particular stage can be associated with a particular age or grade level.

In figure 1 examples of typical responses to the surface development task were shown together with Piaget's stage classification of the responses. Ignoring for the moment the stage classification, do we find that the responses exhibited by the Papua New Guinea sample are similar to those given by Piaget? By referring to figure 2, which shows some typical responses to the surface development task for the four solids used by Piaget by some grade 7 subjects, we can see that there is a wide variation in response to a single solid at a single grade level. This is true also for the other solids used in the task. By comparing these responses to those given in Figure 1 we can observe that there are similarities with some of the responses given by Piaget. As indicated earlier, Anderson (1980) has noted that, for her study with 72 subjects in grades 1 to 6, the responses given yielded a richness in variety not reported in the original study and that this led to difficulties in establishing well-defined stage descriptions. Whilst Piaget does not give details regarding his sample size, it is probably safe to assure that it is smaller than that of Anderson's study and the study reported here. The larger sample sizes of the latter will perforce give rise to a wider variation in responses and this makes it correspondingly difficult to adequately categorize the responses into a sequence of well-defined stages. In attempting to set up stage descriptions, however, the guidelines provided by Piaget with respect to the general features of the drawings do serve as a useful starting-point. Certainly it is not difficult to find responses, given by subjects in this study, which clearly belong to Piaget's sub-stages IIA, IIB and IIIA. From the point of view of this study, difficulties arise when Piaget includes, as part of his stage description, the appearance of correct responses to particular solids.

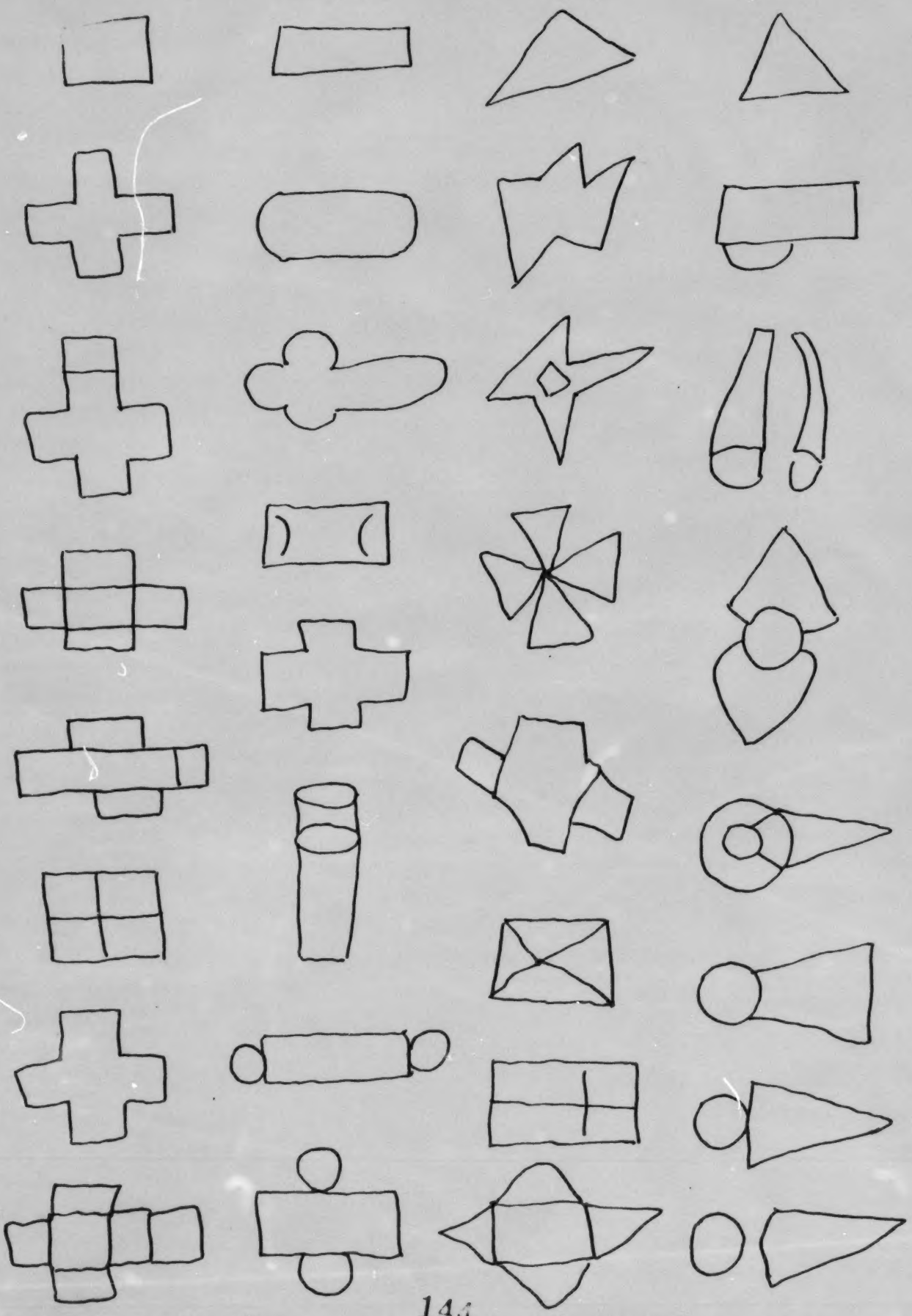


FIG 2: Typical responses to the surface development task for four solids by subjects in grade 7.

As indicated earlier, Piaget found that correct responses were obtained for the cone and the cylinder prior to the cube and the pyramid. Piaget states that: 'In Substage IIIB the correct solutions are arrived at, at least for the cylinder and cone. The cube and the pyramid appear to offer rather more difficulty and a completely correct development of the latter is sometimes not achieved until Stage IV' (Piaget and Inhelder, (1960), p. 277). The criteria for assigning subjects' responses to the higher stages are, thus inappropriate for this study and in order to set up appropriate stage descriptions alternative criteria must be established.

There is a further difficulty associated with assigning a subject's responses to a particular stage. The present study adopted the method of small group testing rather than the typical Piagetian interview technique. Thus only the product of a subject's reasoning, in terms of a set of drawings, is available for analysis and not the additional process of his reasoning, in verbal terms, which also provides data on which to judge a subject's position in the stage hierarchy. This lack of additional information becomes particularly acute in dealing with transitional cases.

For the data obtained in this study, then, a set of criteria was established for assigning a subject's drawn responses to a particular level of competence. The criteria for the lower levels of competence are identical to those given by Piaget in his descriptions of responses which fall into the substages IIA and IIB. The criteria for the higher levels of competence, however, diverge from those given by Piaget with respect to the order of appearance of correct solutions. The criteria adopted are as follows:

Level 2A: As for Substage IIA given by Piaget, i.e. a single feature of the solid such as a rectangle or circle is shown. (See figure 1).

Level 2B: As for Substage IIB given by Piaget, i.e. (see figure 1).

Level 2A/2B: Subjects who displayed both Level 2A and Level 2B responses for their drawings of the nets of the eight solids were assigned to this level. Some subjects, for example, gave a Level 2B response for, say, the cube and pyramid and a Level 2A response for the cone and the cylinder.

Level 3A: As for Substage IIIA given by Piaget, i.e. incomplete developments and partially correct responses. (See figure 1).

Level 2B/3A: Level 2B responses for some solids and Level 3A responses for others.

Level 3B: Correct solutions for the pyramid, cube, tetrahedron and triangular prism. Partially correct solutions for the cone and cylinder.

Level 3A/3B: Correct solutions for at least two of the cube, pyramid, tetrahedron, or triangular prism. Responses to other solids of a Level 3A nature.

Level 4: Correct solutions for the cube, cylinder, tetrahedron, pyramid and cone.

Level 3B/4: Correct solutions for the cube, pyramid, tetrahedron, triangular prism and either the cone or cylinder.

(Almost all subjects had difficulty with the parallelepiped and the pentagonal prism, even those who displayed reasonable competence with the other solids; for this reason they were not included in the competence criteria).

Two research assistants worked independently at the task of assigning subjects' responses to the various levels of competence. In general there was a reasonable degree of agreement; in cases of disagreement the author made the final decision. The results of the classification are set out in Table 9.

LEVEL GRADE	N	2A	2A/2B	2B	2B/3A	3A	3A/3B	3B	3B/4	4
7	77	4	0	27	43	16	10	0	0	0
8	80	1	6	10	21	44	14	1	3	0
9	139	0	4	19	19	27	20	8	1	0
10	85	0	2	5	14	27	32	8	9	2

TABLE 9: Results showing percentages of subjects assigned to a particular competence level, analysis by grade level.

1. At any one grade level there is wide variation in levels of competence.

2. In grade 7, 90% of the subjects showed a level of competence of level 3A or below. The figure for grade 8 is 82%, for grade 9, 72%, and for grade 10, 49%. Thus it is not until grade 10 that we find about half the subjects beginning to display reasonable competence at the task.

3. Over the entire sample only 15% of the subjects gave responses at the 3B/4 or 4 levels of competence.

With respect to sex-differences in performance on the task, it has already been indicated previously that the males' performance tends to be superior to that of the females'. In terms of the quality of the drawn responses it was not apparent that there were any marked differences between the sexes. In comparing levels of competence we refer to the data presented in Table 10:

LEVEL GRADE		N	2A	2A/B	2B	2B/3A	3A	3A/B	3B	3B/4	4
7	F	35	9	0	34	37	11	9	0	0	0
	M	42	0	0	21	48	19	12	0	0	0
8	F	32	3	9	16	19	44	9	0	0	0
	M	48	0	4	6	23	44	17	2	4	0
9	F	48	2	4	27	27	25	10	4	0	0
	M	91	0	4	15	14	29	25	10	2	0
10	F	34	0	6	9	15	32	35	3	0	0
	M	51	0	0	2	14	24	29	12	16	4

TABLE 10: Results showing percentages of females and males assigned to a particular competence level, analysis by grade.

1. The percentage of subjects categorized at level 3A/B or above is, for grade 7, females 9% and males 12%. For grade 8, females 9% and males 23%. For grade 9, females 14%, males 37%. For grade 10, females 38%, males 61%. Thus, at each grade level, the percentage of males performing at the upper levels of competence is greater than that for females, the differences increasing as we proceed from grade 7 through to grade 10.

2. At any given grade level there is a wide variation in the levels of competence at which subjects perform the task. The median level of competence for the males, in each case, tends to be higher than that for females at the grade 9 and 10 levels.

3. A large proportion of the responses of both males and females were categorized into transitional levels of competence. An alternative set of criteria of classification might well be developed such that the nine levels used here could be collapsed into, say, three larger classes, low, medium, and high levels of competence, which may be more useful for descriptive purposes.

4. The median levels of competence for males and females at each grade level are:

   a) grade 7, females 2B/3A, males 2B/3A.
   b) grade 8, females 2B/3A, males 2B/3A.
   c) grade 9, females 2B/3A, males 3A.
   d) grade 10, females 3A, males 3A/3B.

RELATION TO OTHER TESTS IN THE BATTERY

As indicated above, the study involved the administration of a total of seven tests (although not all subjects did each of the seven). The correlation matrices for males and for females are given in Tables 11 and 12. The tests are coded as follows:

S1: Piaget's prediction of cross-sections of solids.
* S2: Piaget's rotation and development of surfaces of solids.
S3: A pencil and paper 'form-board' test.
S4: A visualization/spatial memory task.
S5: A test of concepts of area and perimeter.
S6: Piaget's prediction of loci task.
S7: Elithorn's Perceptual Maze.

Further details of the tests are available in Lean (1980).

TEST	S1	S2	S3	S4	S5	S6	S7
S1	-	.49*	.43*	.24*	.37*	.44*	.37*
S2		-	.31*	.18	.30*	.47*	.39*
S3			-	.19	.28*	.32*	.19
S4				-	.10	.25*	.38*
S5					-	.33*	.25*
S6						-	.11
S7							-

TABLE 11: Correlation matrix for male subjects for the seven tests in the battery. The asterisk indicates correlations which are statistically significant at the .01 level.

TESTS	S1	S2	S3	S4	S5	S6	S7
S1	-	.50*	.34*	.27*	.37*	.43*	.46*
S2		-	.25*	.38*	.25*	.41*	.13
S3			-	.20	.19	.33*	.31*
S4				-	.19	.35*	.37*
S5					-	.24*	.30*
S6						-	.29*
S7							-

TABLE 12: Correlation matrix for female subjects for the seven tests in the battery. The asterisk indicates correlations which are statistically significant at the 0.01 level.

With respect to S2, the rotation and development of surfaces task:

1. For both males and females the highest correlation is between S2 and S1, the prediction of cross-sections of solids. Relatively high correlations exist, for both sexes, between S2 and S6, the prediction of loci task.

2. For males, S2 also has significant correlations with tests S3, S5 and S7. For females S2 also has significant correlations with S3, S4 and S5.

DISCUSSION AND CONCLUSIONS

In replicating a study which is based on Piaget's work, in particular a study which is carried out in a cross-cultural setting, a number of issues need to be addressed. The first issue is whether or not the results of the replication agree with those obtained by Piaget on matters of detail; for example, do we find that the age at which Papua New Guinea subjects can perform a particular task is similar to that quoted by Piaget for his Genevan sample, or is it markedly different? This type of issue is largely of a quantitative nature. The second issue is concerned more with qualitative matters: are the responses given by the Papua New Guinea subjects broadly similar to those given by Piaget's subjects or are they so radically different as to cast doubt on Piaget's theoretical analysis of the task? Any Piagetian task, whilst apparently inconsequential in itself, achieves its significance by association with the theory; any replication may well differ on matters of detail without disconfirming aspects of the theory.

The results of this study, the largest reported in the literature to date, do differ from those of Piaget with respect to detail. As has been found with other Piagetian tasks carried out with Papua New Guinea subjects, there is disagreement as to the age at which subjects can successfully perform the task or perform at a particular level. Faw's (1977) results also disagree with those of Piaget in this respect and, indeed, her results are not dissimilar to those found in this study. Whereas Piaget found that his subjects could successfully rotate or develop the surfaces of the four solids investigated by the age of 12 or 13 years, Papua New Guinea subjects of the same age can manage about one or two; difficulties with the cone and cylinder persist up to 16 years and beyond.

The surface development task is such that the skills required for its successful performance are those which are normally acquired in a school setting, for example drawing skills, the knowledge acquired by viewing and handling various geometric solids together with the associated visualization, imagery, and visual memory skills. Even a child who undergoes a school curriculum that does not specifically stress the geometrical and visual aspect of mathematics may well acquire some of these skills incidentally and thus could display a reasonable level of competence at the surface development task for certain solids. Thus we might expect years of schooling, or grade level, to be a more important experimental variable than chronological age. Indeed, for this study and for Faw's, the difference between mean scores at each grade level tend to be greater than those at each age level. In Papua New Guinea where both the primary and secondary mathematics curricula place relatively little emphasis on the visual aspects of mathematics we nevertheless find that mean performance scores approximately double over the grade 7 to grade 10 range; the median level of competence also increases over the same period. Assuming this to be the result of the incidental acquisition of the skills metioned above, we would expect a greater improvement in performance were the school curriculum to stress the visual aspect of mathematics.

If school experiences do have a bearing on performance of the task we might also expect there to be slight variations between schools in mean performance. To some extent this does seem to be the case although the differences are significant in only a few cases. For example, the mean performance for males at School 5 is significantly lower than that for schools 1, 3 and 4 at grade 7 and 8 levels. At the upper grade levels, however, this is not the case. It is difficult, of course, to pin down the source of these differences; it may have its origin in the differences between the experiences which the schools offer; there may also be certain cultural

schools. Whatever the source, it does appear that the students, both male and female, in grade 7 and 8 at School 5, were disadvantaged in some way with respect to the skills necessary to perform the task. By grades 9 and 10, however, the School 5 students perform at a level which is similar to that of students from the other schools.

Another aspect in which the results of this study differ from those given by Piaget is the order of solids for which subjects can successfully carry out the task. For Papua New Guinea subjects the simpler solids with plane faces such as the pyramid, cube, tetrahedron and triangular prism are mastered prior to the solids with curved surfaces such as the cylinder and cone. With Piaget's subjects the reverse was the case. Faw's (1977) results are, however, in close agreement with those found in this study. In attempting to explain his results, Piaget states that "the cylindrical and conic surfaces are not flat but curved, with the result that the curvature itself tends to suggest the action of unrolling them. Apart from this, these solids have only two a three sides as compared with four or six sides of the pyramid and cube" (Piaget and Inhelder, 1956, p. 286). If, however, experience with various shapes affects performance on the task, it may well be that subjects from the Papua New Guinea island provinces, who are likely to find more shapes with plane surfaces in their environment than shapes with curved surfaces, are able to deal more easily with the former type of solid than the latter. (Although cylindrical shapes are also commonly seen in the form of water tanks and tin cans). It may also be that the skills required to draw the net of the simpler solids with rectangular and triangular faces are of a lower order than those required for the solids with curved surfaces. To draw the net of a pyramid, for example, it may only be necessary for the subject to perceive that there are five separate plane figures to deal with and then to coordinate the relative positions of these figures to one another. In Piagetian terms, this requires

the ability to coordinate perspectives to a certain extent. To draw the net of a cylinder or cone, however, would appear to require the subject to perceive the components making up the solid and then to use the rather more difficult visualization skill of mentally rotating the curved surface to a plane figure and then finally coordinating the relative positions of the resultant plane figures. The difficulty encountered with the pentagonal prism and the parallelepiped may be due not to the perception of the larger number of faces but in coordinating the relative positions of each of the component faces. If we were to consider the task were the subject is given the developed net and then is required to predict what the reconstructed solid looks like, we might expect the subject to require not only the ability to coordinate perspectives but also the visualization skills involved in mentally rotating the component figures. Thus, for solids with plane faces it is possible that the skills necessary to draw the net are less complex than the skills required to reconstruct, by visualization, the solid from the net. Piaget notes that 'It is significant that the children cannot reconstruct the solids simply from a study of the drawings of their development, although they can do so after they have produced such drawings themselves' (Piaget and Inhelder, 1956, p. 286).

With respect to the nature of the drawn responses, the results found in this study agree broadly with those given by Piaget. The variation in responses for a particular solid at a given level of competence tended to be greater for this study than those reported by Piaget and it is likely that this is due to the probable differences in sample size. Also, in this study, a fairly high proportion of subjects gave responses which could be categorized in the transitional levels of competence.

The differences between the results of this study and those reported by Piaget do not necessarily cast doubt on Piaget's theoretical analysis of the task. The age differences

mentioned above could be accounted for by differences in experience which Piaget allows as an important factor affecting performance. The differences between the order of solids which are attempted successfully is not necessary inconsonant with his theory, even if the alternative suggestion, given above, as to how subjects perform the task is accepted, because this explanation and the one given by Piaget both involve the pre-requisite ability to coordinate perspectives. It is also possible, however, that with little or no school experience the ability to draw the nets of solids would not develop at all or would reach a plateau beyond which there is no improvement. This is not to say that unschooled children would not develop certain visualization skills and the use of visual imagery; however the lack of the task specific school skills required to draw the net of a solid would disadvantage then to a great extent.

As indicated in the literature review, Piaget has not concerned himself with the effect upon task performance of such experimental variables as ability and sex. An analysis of sex-differences in performance in this study found that, at all grade levels, the mean scores of males were significantly greater than those for females. Indeed this trend is apparent when we compare mean scores of the sexes over the entire battery of tests (Lean, 1980). In comparing median levels of competence the males in grades 9 and 10 performed at a higher level than the females. The sources of sex differences in spatial ability appear to be complex. On the one hand it has been argued (see McGee, 1979) that there may be genetic reasons and that there may be differences in the extent to which males and females use the right or left hemisphere of the brain in processing information. On the other hand it is possible that the differences could be accounted for in terms of differential experiences associated with sex-roles. If the latter were the case then females may well benefit more than males from additional training and experience in the visual aspects of mathematics.

Faw (1977) devised a task specific two-part training programme for her grade 6 and 8 subjects. In the first part subjects were shown nine solids, different from those in the testing programme, and they were asked to attend to certain features of each solid such as the number and shape of its faces. Subjects were asked to predict what the developed net would look like, after which the solid itself was developed and the drawings were compared to the actual net. This part of the training stressed the use of a reasoning strategy rather than a purely visualization strategy. In the second part of the programme, subjects were shown ten figures each consisting of six connected, numbered squares and were required to predict which of the figures could be folded to form a cube and also to predict what numbers would be on the faces that would be opposite each other when the figure was folded. The subject was then allowed to fold the figure to check his prediction. The results of the training significantly improved performance on a test requiring the subjects to draw the predicted nets of a selection of other solids (there did not appear to be much of a transfer effect to the other tests used however). The use of a similar type of training program may well have similar beneficial effects with Papua New Guinea school children.

It is probably true to say that both the primary and secondary mathematics curricula emphasize the analytic and algorithmic aspects of mathematics at the expense of the visual, spatial or geometric aspects. Lean and Clements (1981) found that tertiary students in Papua New Guinea tend to be analytic rather than visual problem solvers. The use of visualization skills and visual imagery in problem solving have often been reported by mathematicians and scientists. The possession of such skills and the ability to use them effectively provide an additional string to the problem-solver's bow. On this basis alone an argument may be made to increase the visual aspects of mathematics in Papua New Guinea curricula: the frequent use of diagrams to represent and summarize mathematical

ideas, increasing practical activities associated with the study of area and volume, the introduction of activities involving the construction of plane and solid figures, activities associated with determining the locus of a moving point. Such a programme would serve not only to increase students' general knowledge but, by a marriage of visual skills with the analytic, serve to increase the versatility and range of their problem-solving skills.

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ATTITUDES TO SUCCESS AND FAILURE IN MATHEMATICS

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A B S T R A C T

Attitudes to success and failure in mathematics.

The extent to which affective factors influence learning is an important question for educators. In this paper, some relevant research is summarised and recent Australian data are reported. These data indicate how students and teachers react to success and failure situations in mathematics. Some differences in attributional patterns were found between high and low achievers in mathematics, and between males and females. The role of teachers in shaping both functional and dysfunctional attribution patterns is discussed. The question of the appropriateness of generalizing findings across cultures is raised.

> Nobody starts off stupid. You have only to watch babies and infants, and think seriously about what all of them learn and do, to see that, except for the most grossly retarded, they show a style of life, and a leisure and ability to learn that in an older person we might well call genius..... But what happens, as we get older, to this extraordinary capacity for learning and intellectual growth?
>
> What happens is that it is destroyed..... by the process that we misname education.... we destroy this capacity above all by making them afraid, afraid of not doing what other people want, of not pleasing, of making mistakes, of being wrong.
>
> (Holt, 1976, p.165)

In recent years, interest has increasingly focussed on the extent to which affective factors influence learning. This concern is also reflected in the present paper in which three separate studies are described. Two of the studies were concerned with the way students react to success or failure, particularly in mathematics, while in the third study an attempt was made to monitor student teacher interaction. To allow these studies to be seen in perspective, some relevant previous research is briefly described below.

In a large number of laboratory studies conducted with animals it was found that a variety of organisms exposed to events in which the outcomes were not related to responses made by them, frequently exhibited subsequent disruption of behaviour. For example, whereas naive dogs efficiently learned to escape shock by jumping over a barrier in a shuttle box, dogs that first received shocks they could neither avoid nor escape showed marked deficits in acquiring the shuttle escape response (Seligman & Maier, 1967). Similarly debilitating behaviour has been reported in experimental work with cats, fish and rats (see Abramson, Seligman and Teasdale, 1978 for a review of this research), and more importantly for the educator, in humans. In all these cases, 'learned helplessness' has been used to denote the expectation that outcomes are non-contingently related to one's own responses. There is some evidence,

(Dweck, 1975) that with children, the belief that failure is insurmountable may be accompanied by a deterioration in performance so that some become incapable of solving the same problems, that prior to failure, they could solve with ease.

The theoretical model most frequently used to help understand and explain performance following success and failure is the one proposed by Weiner (1972, 1980). In it, attributions and perceptions of success and failure are categorized along a number of dimensions: locus (internal v. external), stability (stable v. unstable), and controllability (controllable v. uncontrollable). The perceived stability and controllability of the attributions, as well as the extent to which they are regarded internal as opposed to external, are all considered to be important determinants of future effort and expectations. If, for instance, failure on a mathematics test is attributed to an unstable cause (e.g., that particular mathematics test was very difficult, it was too noisy that day to concentrate properly) then failure can presumably be avoided in the future, and subsequent efforts to attain success should not be affected. If, however, failure is attributed to a stable cause (mathematics teachers always give unfair tests, I can never do the problems set in mathematics tests), then the individual may come to believe that failure can not be avoided in the future, that he/she is unable to determine future outcomes, and that therefore there is not much point in trying hard. The perceived reasons for success are equally important. If success is attributed to an external and unstable cause, success is not assured in the future and similar tasks may be avoided. Attribution of success to an internal and stable cause such as ability, on the other hand, would lead to expectations of future success and continued efforts on similar and related tasks. Generally, functional attributions are hypothesized to lead to continued or increased efforts, while inappropriate attributions of success and failure may lead to cognitive, motivational and/or emotional deficits. One such attribution and resultant chain of events has been summarised by Weiner (1980) as shown in figure 1.

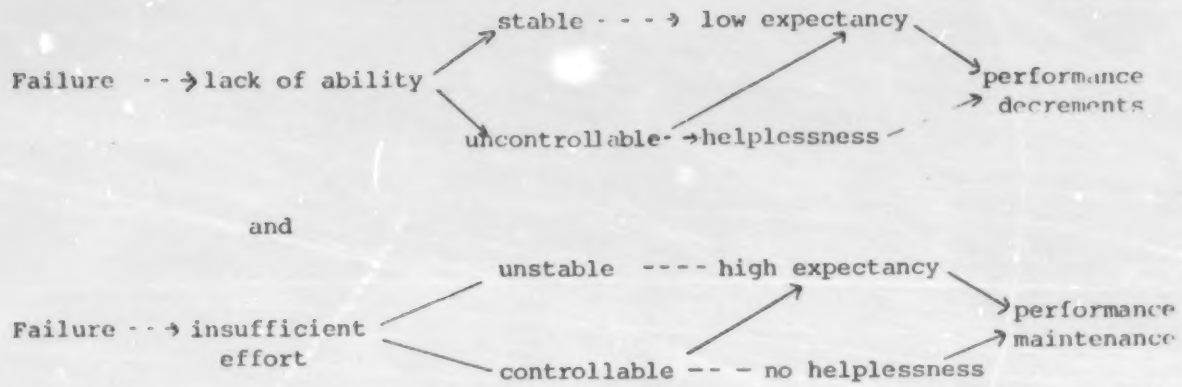

stable - - - → low expectancy
Failure - - → lack of ability
performance decrements
uncontrollable → helplessness
and
unstable ---- high expectancy
Failure - - → insufficient effort
performance maintenance
controllable -- - no helplessness

Figure 1: Attribution and hypothesized outcome.

It is easy to understand why Weiner's causal attribution model has seemed so relevant to educators concerned about students who exhibit learned helplessness-type behaviour. Anecdotal descriptions of such students abound.

> The first thing people remember about failing at maths is that it felt like sudden death. Whether it happened while learning word problems in sixth grade, coping with equations in high school, or first confronting calculus and statistics....., failure was sudden and very frightening. An idea or a new operation was not just difficult, it was impossible! And instead of asking questions or taking the lesson slowly, assuming that in a month or so they would be able to digest it, people remember the feeling, as certain as it was sudden, that they would never go any further in mathematics.
>
> (Tobias, 1978, p.44)

Weiner (1980) has argued that how teachers react, both verbally and non-verbally, to failure by their students helps to determine whether such students ascribe their failure to lack of effort or lack of ability. If a teacher is sympathetic, and appears to be comforting a student presumed to need such support, "such affects are likely to exacerbate feelings of incompetence because they imply low ability attributions. And sympathy might prevent the teacher from making demands on the student, or expecting high performance. Sympathy also may lead to too much help, taking responsibility away from the student" (p.10). If on the other hand the teacher assesses or implies that failure is due to lack of effort, then the student is likely to feel guilty, the teacher angry and excessive help tends to be withheld so that the student is forced to accept responsibility. Thus the teachers' attributions of their students' success and failures reflect their expectations of the future success of those students. 166

When carefully manipulated, failure, it seems, can become a cue to escalate effort. Qualified support for this contention is offered by Relich (1981). Interestingly enough there is also some experimental evidence (Dweck, 1977) that learning situations that involve errorless learning may render children less able to deal with subsequent errors).

One of the studies reported here examines how pre-service teachers react to their own success and failure in mathematics. The other two studies examine school students' perceptions of their success and failure. Before describing these studies and their findings one further factor needs to be discussed.

There is a substantial body of evidence to indicate that dysfunctional attributional patterns likely to lead to learned helplessness are more prevalent among girls than boys. Of particular interest are the findings reported by Wolleat, Pedro, Becker and Fennema (1980) of sex differences in the attributional patterns for success and failure in mathematics. In their sample of over 1200 high school students, success was attributed to ability more frequently by males than females. Females were more likely than males to attribute success in mathematics to an unstable dimension (effort), but failure to stable dimensions (ability, task difficulty). Even when their performances on a mathematics achievement test were similar, the strengths of their attributions differed. For example, the males' success ability scores were found to be higher than those of the females at all levels of achievement. In contrast, when responding to failure events, the females' attribution to lack of ability was stronger than that of males, again at every level of achievement.

To facilitate comparison of the results of the studies carried out in Victoria, Australia, in 1981, with the data obtained with American samples, where possible the method and instruments used in the former closely resembled those employed by Wolleat et al.(1980). Emphasis was on the locus and stability dimensions, rather than the controllability dimension.

Study 1.

Method

The Sample: The sample comprised 94 secondary school students, 48 males and 46 females, in grades 7 and 8. They came from four classes at three co-educational high schools in the metropolitan area of Melbourne.

The Instrument: The students were asked to complete a Mathematics Attribution Scale (MAS), closely modelled on the one used by Wolleat et al. (1980). The MAS contains eight subscales of four items each, and allows measures of success and failure attributions with respect to ability, environment, effort and task to be obtained. One set of items is shown in figure 2.

Event 6. You were able to understand a difficult section of the mathematics course.

(1)	The way the teacher presented the topic helped	SA	A	U	D
(2)	Your ability to do maths is more obvious when you are challenged.	SA	A	U	D
(3)	You put hours of extra study into the topic	SA	A	U	D
(4)	The problems were very easy because the work had been covered before.	SA	A	U	D

* Subjects were asked to circle the most appropriate response.

Figure 2: Items from the MAS scale.

In addition, the students were ranked by their mathematics teachers on the basis of their mathematics performance in class.

RESULTS

To determine whether or not there are sex differences on the MAS, t tests were carried out. As well, multiple regression was used to examine, separately for each of the classes tested, relations between attribution patterns, sex, and achievement level in mathematics. The results are not reported in full, to conform with space allocation.

Statistically significant differences between males' and females' responses were found on three of the eight subscales. Males seemed to attribute success in mathematics to ability more strongly than females ($p < .001$), while females seemed to attribute failure in mathematics more strongly than males to ability and effort ($p < .07$ and $p < .02$ respectively).

The 32 multiple correlation coefficients ranged in value from .27 to .71. Thus the total amount of the MAS scores explained by sex, mathematics achievement level, and their interaction ranged from just over 7% (on the failure task scale for class 2) to approximately 51% (on the success ability subscale for class 1). A more detailed look at the results for one class will illustrate how the causal attributions of success and failure events varied by sex and achievement level. For this class statistically significant portions of the variance ($p < .05$) were explained by sex on the success ability and failure ability subscales, and by sex level of achievement on the success task, failure environment and failure task subscales. The data are shown in figure 3.

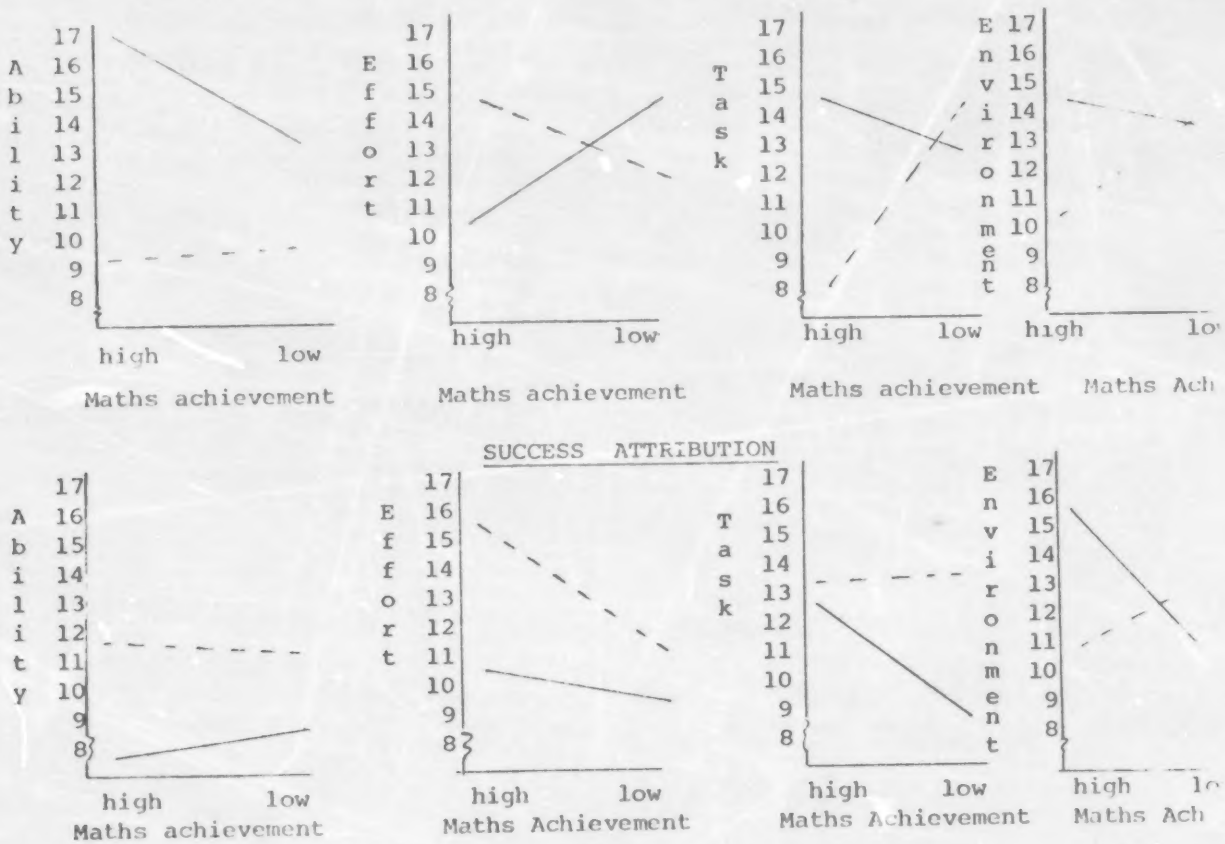


Figure 3. Causal attribution of success and failure.

Of particular interest are the following:

- At every level of achievement the boys attributed success in mathematics to ability more strongly than did the girls, while at every level of achievement the girls attributed failure in mathematics more strongly to ability than the boys.

- As well the mathematically capable boys (in terms of their achievement in class) attributed success in mathematics more strongly to task and less strongly to effort than did the mathematically capable girls.

- The mathematically capable girls tended to attribute their failure in mathematics more strongly than the boys not only to ability but also to effort and task.

It can be seen, then, that in general the attributional patterns of success and failure in mathematics of the boys and particularly of the mathematically achieving boys tended to be more functional than those of the girls, and particularly of the mathematically achieving girls.

Dweck (1977) has argued that teachers may unwittingly be contributing to the different attributional patterns of boys and girls because of the way they interact with their male and female students. Weiner (1980) has also stressed the importance of teacher-student interactions in reinforcing functional or dysfunctional attributional patterns. The second study reported here examines how a group of pre-service teachers attribute their success and failure in mathematics.

Study 2:

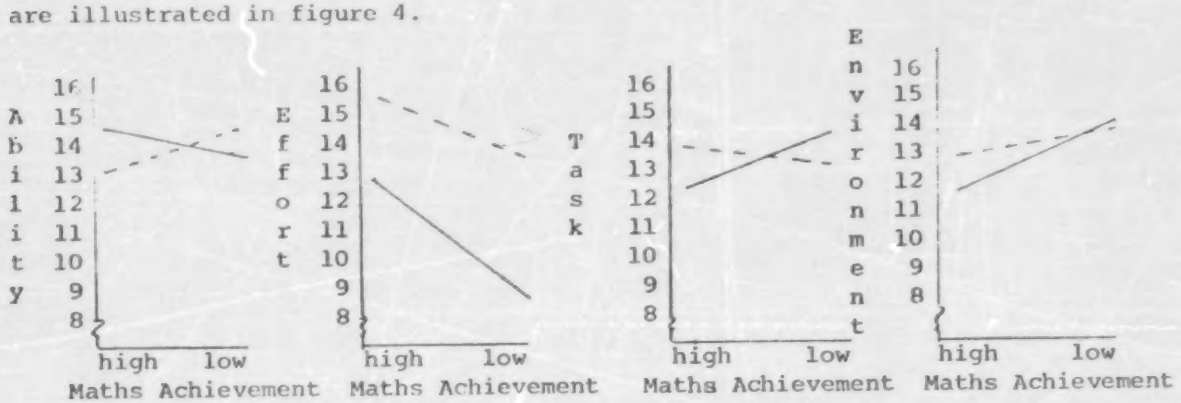
METHOD

The Sample: This time the sample comprised 41 pre-service teachers, 19 females and 22 males. All the subjects had taken sufficient tertiary mathematics units to be eligible for eventual employment as mathematics teachers in secondary schools.

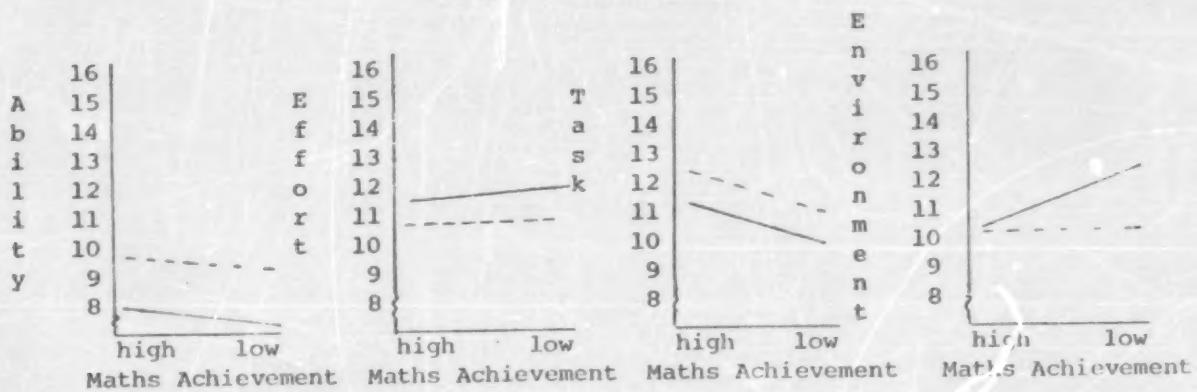
The Instrument: The subjects were asked to complete the MAS scale. They were also asked to give details of the number of tertiary mathematics courses taken and their performance in each of these units. A mathematics proficiency rating that took account of both these measures were computed. The analyses performed were similar to those described for the previous study.

RESULTS

The eight multiple correlation coefficients obtained ranged in value from .15 to .61. Thus the total amount of variance on the MAS scale explained by sex, mathematics achievement level as previously defined, and their interaction ranged from approximately 2% to 38%. On the success effort scale, statistically significant differences were found for both sex and mathematics rank ($p < .05$ and $p < .001$ respectively), while on the failure task subscale a statistically significant difference was found for mathematics achievement with high achieving students attributing failure to task more strongly than lower achieving students ($p < .05$). Furthermore, the males in this sample rated ability a more important factor in attaining success in mathematics but less important in failing mathematics than did the females in this sample, though the difference failed to reach statistical significance ($p < .20$ in each case). These data are illustrated in figure 4.



SUCCESS ATTRIBUTION



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FAILURE ATTRIBUTION

Figure 4: Causal attributions of success and failure events.

It is important to remember that all the subjects in this sample were mathematically proficient. Those with the lowest mathematics rating score had still satisfactorily completed a number of tertiary mathematics units. For them failure was presumably relative, rather than absolute. Nevertheless, there was a statistically significant difference between male and female pre-service teachers in the degree to which success in mathematics was attributed to effort. Some evidence was also found for other sex differences in attributions described in study 1. As well, the more successful mathematics students were more likely to attribute their success in mathematics to effort and their failure to task than the less successful students. Thus the more successful students tended to have a less functional pattern of attribution than the relatively less successful students. Is it inappropriate to speculate if these differences in attributional patterns will be conveyed to their students? The last study reported here is concerned not only with attributions of success and failure, but also with student teacher interactions.

Study 3*

METHOD

The Sample

Twenty-four students from two classes in a special school in the metropolitan area in Melbourne, Victoria, comprised the sample. Both classes contained twelve students and were taught by female teachers. The students were aged between 14 years 4 months and 16 years 4 months at the time of testing (October, 1981) There were 10 males and 14 females.

The Instrument: Use of the MAS was inappropriate for this sample. Instead a modified and simplified version of the scale used by Crandall, Katkovsky and Crandall (1965) was used to ascertain success and failure attribution. The final scale contained 12 items, six depicting failure situations and six describing success situations. Two possible responses were provided for each item. Functional success and failure attribution scores could thus be calculated for each student. (In line with the theoretical framework outlined before, internal attributions for success events but external

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* For justification for the type of sample used in this study, see Leder (1981) The data reported were gathered as part of a study carried out together with L. Bartak and R. Ritli . Their help is gratefully acknowledged.

attributions for failure situations were considered functional.) To maximize the relevancy of the items to the students in this sample, eight of the items were of a general nature, while only four items were concerned with success and failure in mathematics and reading. One of the items is shown in figure 5 below.

When you learn something quickly at school, is it because -

a. (Your teacher) showed you

or b. Because you listened carefully.

Figure 5: An item from the attribution scale.

The scale was administered individually to each student who was expected to give an oral response. Several items were administered twice, with the order of the responses provided reversed to check for student consistency. Students were also given a mathematics test: the Basic Number Screening test (Gillham and Hesse, 1976). In addition, for nine one hour periods in one class and seven periods for the other class, the number of student-teacher interactions was recorded. A distinction was made between directives and any other interactions.

RESULTS

The data concerned with student-teacher interactions only are reported first. For ease of presentation they are summarised in Tables 1 and 2.

Table 1: All interactions

	Interactions	Directives	% of directives
Class 1	501	115	23.0
Class 2	596	244	40.9

Table 2: Student-teacher interactions, with individual students only.

	Interactions	Mean (males)	Mean (Females)	(directives x 100)% interactions	M	F
Class 1	310	22.8	27.4		5.5	24.7
Class 2	514	44.0	41.7		37.5	48.4

While there were clear differences in the extent to which the two teachers interacted with their class, no appreciable differences were found in the number of times they interacted with their male and female students. However, differences were revealed when the type of interactions were considered. In both classes, considerably fewer of the interactions with boys consisted of simple directives. Because of the small number of students in each class, these data need to be interpreted with caution.

Small but positive correlations (.23 and .43) were found between the students' performance in the mathematics test and student teacher interactions. Both teachers, it seems, tended to interact somewhat more with the relatively successful students.

The correlations between teacher-student interactions and functional attributions are presented next, in Table 3.

Table 3: Correlations between functional attributions and student-teacher interactions.

Correlation between	Class 1 (N=12)	Class 2 (N=12)
	ρ	ρ
success attrib^{n}. & interactions	- .31	- .15
failure attrib^{n}. & interactions	- .32	- .43

In both classes, there were fairly small but negative correlations between the number of interactions with the teacher and the functional attribution of success and failure events. Thus in both classes, the higher the number of interactions with the teacher, the less functional tended to be the students' attributions of success and failure.

Finally, it is of interest to examine the relation between performance in mathematics and functional attributions. The data are presented in Table 4.

Table 4: Correlations between functional attributions and performance in mathematics.

Correlations between	all students	M(N=10)	F(N=14)	Class 1	Class 2
	ρ	ρ	ρ	ρ	ρ
success attrib. & maths	.41	.14	.56	.52	.19
failure attrib. & maths	-.32	-.46	-.19	-.25	-.39

There is a consistency in the pattern of correlations obtained. In each group the mathematically more successful students were more likely to perceive themselves as being responsible for outcomes of events, irrespective of whether the events depicted success or failure situations.

Summarising these results, for the students in this sample, there was a positive correlation between the number of student-teacher interactions and the students' performance on the mathematics test, as well as between the number of student-teacher interactions and the extent to which students perceived themselves responsible for failures but others for success events - a dysfunctional pattern of attribution. The students' mathematics performance was positively correlated with the extent to which they internalized responsibility for the outcome of events - both success and failure. Only for success events is internalizing of responsibility regarded as functional, however. The teachers differed in the type, but not the number of interactions they had with their male and female students.

CONCLUSIONS

The findings of three small studies have been presented. The results of the first study indicated that the attributional patterns of secondary school boys for success and failure in mathematics tended to be more functional than those of the girls in the sample. The relatively dysfunctional attribution of success and failure in mathematics shown by the sample of pre-service and particularly the females, described in the second study was disturbing and raises questions about the extent to which such dysfunctional attributions may be conveyed to students. In the third of the small studies reported, the sample comprised students attending a special school. Only a very simple instrument for assessing attributions could be used. Relations between student-teacher interactions, mathematics performance and attributions of success and failure events were examined. Positive correlations were found between the number of interactions and the students' performance in mathematics, but the benefit of frequent interactions with the teacher was not reflected in the students' pattern of attributions. Those who interacted more with their teacher had a less functional pattern of attribution than those who had fewer interactions.

Because of the small size of the sample, few analyses could be performed separately for each sex. Nevertheless, it appeared that the teachers differed in the type, but not the number, of interactions they had with their male and female students. Finer distinctions in the type of interactions taking place could appropriately be considered in future studies.

It would be presumptuous of me to suggest that the results from Australian schools with Australian students can be translated without alteration to the Papua New Guinea situation. But it does seem that the findings present possible food for thought. Are there certain types of students in Papua New Guinea - perhaps girls, perhaps low achievers, perhaps another sub-group of students - whose attributions of success and failure in mathematics are inappropriately dysfunctional and mitigate against continued satisfactory mathematics learning? How can such students best be helped by their teachers? Perhaps local research will provide the answer.

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COMMUNITY SCHOOL MATHEMATICS IN PAPUA NEW GUINEA: THE INTERNATIONAL CONNECTION

By

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COMMUNITY SCHOOL MATHEMATICS IN PAPUA NEW GUINEA:

THE INTERNATIONAL CONNECTION

THIS STUDY

This paper presents findings from an exploratory study done in 1977 and 1978 as part of a larger study of standards in community school mathematics education (Roberts 1981). The study is about how well Papua New Guinea children in international schools do in Mathematics compared with their contemporaries in Community Schools.

BACKGROUND

In the main, two systems of schooling operate in Papua New Guinea, the largest being the system of schools for Papua New Guinea children which for convenience in this paper I shall call the "national system" the other being the "international" system- operating also within the control of the National Department of Education and catering essentially for the children of foreigners working in the country. There are 2118 community schools and 102 provincial high schools in the national education system compared with 32 primary and 2 secondary schools in the international system. (Planning Services Branch 1981.)

The international schools follow a similar curriculum to that of Australian state schools.

Papua New Guinean parents who can afford the fees-approximately K1000 ($(A)1300) per annum for primary and K2000 p.a. for secondary compared with K10 p.a. for the average community school, and K150 p.a. for provincial high schools-may enrol their children into international schools.

Educational provision in these two systems differs and there has been mention recently (Tulo 1982) of raising the quality of schooling in community schools to that provided by the international system. The assumption being that "international schooling" as provided by PNG's international schools is better for PNG children than that provided in the national schools. There are many aspects of this question which need to be examined. However, leaving aside the questions of the economics, relating to materials and teacher education provision, cultural concerns, and so forth, this study simply attempted to look at these questions:

1. Do PNG children in international schools do better at Mathematics than their community school contemporaries?

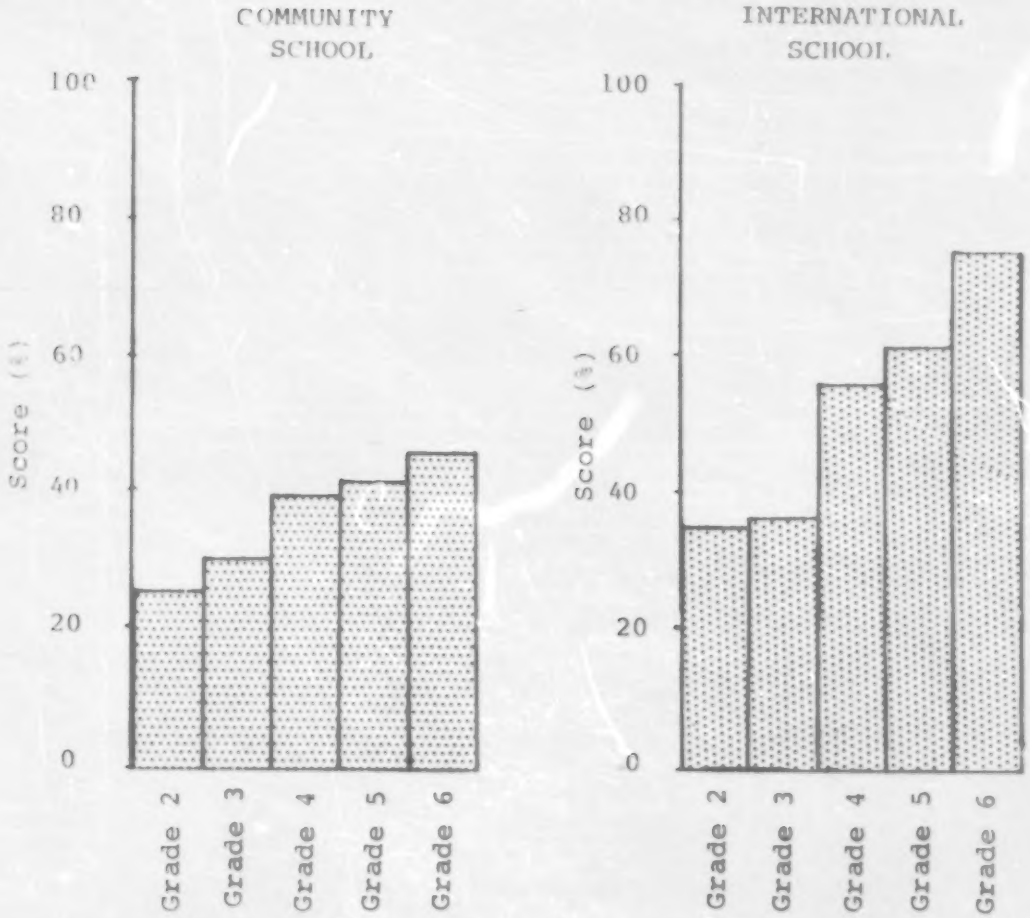
2. How do PNG children in international schools perform (in Mathematics) compared with their peers within that school system?

At the time of this study an average of 10% of children in international primary schools were Papua New Guinean, with more like 15% in the lower grades.

METHOD

Assuming that the mathematics taught in community schools (MaCs) Curriculum Unit 1976)was a subset of that taught in the international schools, a test of general mathematics achievement (copy attached) was given to 85 teachers in a selection of rural, remote rural and urban community schools, 15 teachers in an international school, 967 randomly selected children in 23 community schools, and to 87 children in 3 international schools.

FIGURE 1: Comparison Between Community School Students and International School Students' Performances on the MaCS Achievement Test.



	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
	187	187	187	198	196
$\bar{x}$(%)	25.3	30.3	39.2	41.5	46.8
s	4.9	8.6	4.2	5.7	7.1

	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
n	6	10	10	10	10
$\bar{x}$(%)	35.1	36.4	56.7	61.0	75.1
s	5.6	8.5	14.0	11.8	8.3

This latter figure includes a subset of 41 PNG children. PNG children means children whose mothers and whose fathers are PNG nationals. The teachers and the children did the same test. Children from Grades 2 to 6 were tested. The questions were read to the Grade 2 children. The test contained 60 items, 12 items for each grade level. Children were encouraged to answer as many questions as they could.

RESULTS

Figure 1 compares the scores of children from the national system with those from the international system.

Schooling and maturation effects were clear in both cases. There were relatively small differences in the scores of the Grade 2's in the national system and those of the Grade 2's in the international system. Similarly with the Grade 3 scores. This is interesting since children in international schools generally have had the benefit of pre-schooling, apart from a home environment likely to support the development of mathematical concepts. This small difference may indicate how well community school teachers present the Grade 2 and 3 MaCS material. This author's personal observation of community school classroom work in those grades supports this notion.

Substantial differences begin to occur, however, at Grade 4 where children in international schools appear to reach a "take off" point. Thereafter, the developmental trend, while remaining linear for community school children, begins to become exponential for children in international schools.

This would appear to be consistent with Dr. Randall Souviney's findings (Souviney 1981:128). Souviney observed:

"We suspect that in the early school years children rely on their well developed ability to memorize visual input (Bishop 1979, Lean and Clements 1980) while developing basic English skills. Over the next few years those children that reach some as of yet unidentified threshold of language competency excell in mathematics learning as well, while those remaining dependent on visual memory for processing mathematical input are overwhelmed with the vastly increasing symbolic load (my emphasis)."

It would seem reasonable to examine the mathematics learning/language interface as the most likely and major factor in children's attainment of this "take off" level. It is interesting to note the delayed take off point for the PNG children after Grade 4, compared with their foreign counterparts (after Grade 3). However, the sample size is extemely small. Clearly then children in the international schools beyond the Grade 3 level were mathematically well advanced on those in community schools.

Table 1 presents the comparison of teacher scores on the same test.

TABLE 1: Comparison of Scores of Community School Teachers and International School Teachers on the MaCS Achievement Test.

School	n	$\bar{x}$ (%)	s	t	df.	P
Community	85	78.1	9.7	6.0	98	.001
International	15	93.4	3.9			

Although it might be expected that teachers in both systems would have near full marks on the test, community school teachers fall short of that expectation. The test does not give international school teacher full scope to demonstrate what they actually do know of Mathematics. What is relevant here is that on this test of the content of MaCS programme international school teachers knew significantly more mathematics than their community school counterparts. The implications of this are obvious (note Beeby 1966) and this is also a major factor in determining differences in standards.

Table 2 compares the scores of PNG children in international schools with those of their fellow students.

TABLE 2: Scores on MaCS Achievement Test: PNG Students Compared With Their Fellow Students in International Primary Schools

Grade	PNG Children			International Overall					
	n	$\bar{x}$ (%)	s	n	$\bar{x}$ (%)	s	d.f.	t	P
2	16	30.9	10.3	6	35.1	5.6	20	0.94	n.S.
3	6	49.6	7.1	10	36.4	8.5	14	3.18	0.01
4	4	44.7	8.1	10	56.7	14.0	12	1.59	n.S.
5	7	59.4	6.0	10	61.0	11.8	15	0.39	n.S.
6	8	67.4	4.9	10	75.1	8.3	13	1.894	n.S.

NB: PNG children were tested in 3 international primary schools. The overall figures came from 1 of these international primary schools.

It would appear that PNG children in the international schools were able to keep up with their international fellow students. Of course, this PNG cohort of students was likely to be a rather special group at that time. Most of them would have come from homes where parents were educated, relatively well off, spoke English often, and were education conscious.

Finally, Table 3 brings us to the direct comparison of children in community schools with PNG children in the international primary schools.

Except for a discrepancy at the Grade 4 level which is most likely a result of the small sample size, PNG children receiving mathematics instruction in the international schools were learning significantly more Mathematics than their brothers and sisters in the community schools.

This answers the first research question. The second question was answered earlier with Table 2 showing that PNG children in international schools appeared generally to keep pace with their foreign schoolmates.

The proportion of PNG students in international schools has risen since 1977 to something like 15-20%, with higher proportions, perhaps 20-25%, in the early grades. Replication and extension of this study would most likely provide more valid data and more specific trends than this study can provide.

RECOMMENDATIONS

It is this author's belief that since the opportunity exists to study the education of PNG students under certain conditions considered desirable by the Ministry of Education, all possible

TABLE 3: Comparative Scores of PNG Children in Community Schools with PNG Children in International Schools on the MaCS Achievement Test.

Grade	Community Schools			International School			d.f.	t	P
	n	$\bar{x}$	s	n	$\bar{x}$	s			
2	187	25.3	4.4	16	30.9	10.3	201	4.22	.001
3	187	30.3	8.6	6	49.6	7.1	191	5.44	.001
4	181	39.2	4.2	4	44.7	8.1	183	2.53	n.s.
5	198	41.5	5.7	7	59.4	6.2	203	8.14	.001
6	196	46.8	7.1	8	67.4	4.6	202	8.13	.001

should be done to determine which aspects of the international education system most benefit PNG students. Attempts need to be made to identify which of these aspects could be replicated in community schools.

The following research question may provide a basic for such a study:

To what methods and materials do PNG students in international schools best respond? (Particular emphasis should be placed on successful language acquisition, particularly at the infant level.)

As PNGs' national education system expands and hopefully delivers a better service this opportunity to gain from the presence of such "model" schools needs to be maximised.

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A CRITIQUE ON RANDALL SOUVINEY'S APPROACH TO TEACH NUMBER FACTS IN GRADE 7

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A CRITIQUE ON RANDALL SOUVINEY'S APPROACH TO TEACH NUMBERFACTS GRADE 7

INTRODUCTION

I would like to start this paper with a quote from my paper on Transition Rates and Mathematics Achievement in Five Developing Countries (Ros 1980).

"It is contended that in Papua New Guinea confused thinking on the proper aims of Secondary Education, the increasing transition rate, inappropriate methods of assessment, and the quota system have all contributed to the prevailing low standards. The high transition rate, especially, has resulted in students of a very wide range of ability being admitted into secondary schooling, and this combined with current educational practice in PNG makes effective teaching very difficult. The present educational system panders to students of low ability at the expense of retarding the development of the more gifted. Unfortunately it is just this group from which society has to draw its expertise and leadership in succeeding generations. If this group fails to find a challenge in education, gets bored, and loses its motivation, the consequence is a brood of potential trouble-makers. The problem is compounded by the fact that an increasing mass of frustrate unemployed is being generated. The Education Plan 1976-1980 admits that "there has been over-expansion and some degree of distortion in the post-primary sector" (a gruss understatement), and adds that "the social demand for new high schools is so strong (reinforced by official promises) that it is not feasible to stop high school expansion" p. 124. If this is in fact the case then the prospects of improving standards are very dim indeed."

The conclusion of that paper could become the introduction of any paper on Mathematics education. I believe that educational attainment of students in PNG has been severely compromised by the schooling students receive in the Primary and Secondary schools because of reasons mentioned above. I do not believe as some other people do, that "cultural backwardness" is the reason for poor performance.

Concern about standards of the Grade 10 output and the Grade 11 intake was also expressed in some papers on 'The Rogers' Proposals' (doubling the number of National High Schools as a matter of urgency.) which I distributed last year during the conference in support of my paper: Standards: A Defeatest Point of View.

This concern was again expressed during the closing session of last year's Maths Workshop in Goroka. I had found it necessary to voice this concern, since text-books were going to be written without in my opinion, clear objectives in mind as to what to cater for and what to achieve. I questioned whether it is at all possible to cater for the wide range of abilities by using one text-book.

This concern stems from the fact that at Goroka Teachers' College we have to teach content and method in a responsibe way.

GENERAL COMMENTS

In order to determine what responsible teaching is, we have to know very clearly what the aims of secondary education are. The aims as stated in PNG National Education Strategy 1979, p.35 are incompatible and attempts to implement them have contributed to low standards and to confusion concerning the content ofthe Provincial High School syllabus. During the meetings of the Provincial High School Mathematics Syllabus Review Committee comments such as: This is too difficult; this is irrelevant; my kiddies (Chuave) don't need this, were often head.

It should be noted that the approach suggested by Randall Souviney was intended and written for Community Schools. After the Syllabus Review Committee had approved Murray Britt's material, and after the Grade 7 and 8 booklets had been trialled, Souviney's approach (The concrete, pictorial and abstract) was introduced in the Grade 7A book. Neither the Syllabus Review Committee nor Goroka Teachers' College had any knowledge of such changes which were to be introduced. The result is that Sauviney's approach which was as I said intended for grades 2-6 is now condensed to a 2 months course in Grade 7.

COMMENTS ON TEACHING CONDITIONS

It should be born in mind that the given material in most cases will have to be taught by hopefully enthusiastic but inexperienced Goroka graduates, who would find themselves more familiar with the traditional stuff than with the new Grade 7A material.
Secondly, that the material will have to be taught in packed classroom conditions, where often forty or more lively kids, full of energy, sit crammed together.
Thirdly, that often materials which abound at Goroka Teachers' College are usually scarce or non-existent in most schools.
Furthermore, a large number of students will still be suffering from a number-base syndrome as a result of Brian Norris' approach on 'Learning Numbers'. This approach was introduced by the curriculen unit in 1977 and dominates the Community School Mathematics Syllabus for Grade 4 and 5.
I would like to invite any member of the audience to try and teach what book 7A suggests under the above mentioned conditions.
Teachers' confidence, interest, job satisfaction and morale are often established (or not) during the first weeks of his teaching career. If the teacher fails in this, it will be a disaster for the rest of his career.
In view of all this, one should consider the teacher's state of mind when he is about to start chapter 3 'Working with Numbers' after he has struggled through the initiation ceremonies (Making Patterns and Counting Systems).

SPECIFIC COMMENTS ON THE INTRODUCTION OF RANDALL SOUVINEY'S APPROACH IN THE GRADE 7A BOOK (MATHEMATICS OUR WAY).

A. There was no need to introduce a new approach. Teachers in PNG have always managed quite well to teach natural numbers and the four operations.

(i) Peter Sullivan (1981) commented on test results (PNG, grade 10; U.S.A. 17 yrs/old; Australia 14 yrs/old). He says: These results, rather than causing alarm at levels of arithmetic skills, when compared with results from U.S.A. and Australia reveal that students here may be doing marginally better. The multi-digit division problems are perhaps lower than one would hope, but again PNG students are doing better than their counterparts. Of the errors students make, many can be contributed to carelessness.

(ii) My own data collected since 1976 show that PNG students have always been performing reasonably well in naturals and decimals with the operations + and -, and to a lesser extend in the multiplication and division of Naturals errors are due to lack of drill, carelessness and underestimating e.g. in $95\overline{)\ 67\ 355}$ (correct answer 709) many answers such as 79 or 6109, 7081, 781 etc are given.

(iii) An analysis of my D_2 test administered to the Grade 11 intake shows that 76% (sample 784) can do 0.1 - 0.01, 75% can do $2.16 \div 3$ and 59% can do $0.16 \div 4$ and $12.35 \div 13$, but only 40% can do $8.464 \div 0.8$.

B. Souviney's aim at 100% understanding via the concrete, pictorial, abstract approach is unrealistic. From my experience full understanding comes gradually, There is no need to explain algorithms in detail. If students understand simple cases, a generalisation should suffice.

C. Souviney's approach will most probably be introduced in the Community schools (Grades 3 to 6). It will also be used in Grade 7. Over that period students will likely be taught by at least 5 different teachers. Some of these teachers will refuse to teach the new methods. One can imagine the confusion this new approach will create.

D. Given the present class-room conditions, the inexperienced teacher, the financial constraints, the success of the suggested approach is very dubious to say the least.

COMMENTS ON ADDITION AND SUBTRACTION

I won't argue about the regrouping method suggested in 7A. It is obvious that regrouping is superior than the borrowing or pay-back method. The pictorial illustrations will be helpful.

COMMENTS ON MULTIPLICATION (See appendix A)

Reference is made to the example given: 13 x 15 and the teaching suggestions. If we take the suggestions seriously

a. to use base 10 materials
b. board
c. chalk
d. squared paper
e. pencil

it would take, I would imagine, 40 minutes of arduous work to just go over one example using the concrete, the pictorial and the abstract, explaining the set up and laying all the connections.

Reference is made to the pictures which show the multiplications: 30 x 40 and 27 x 63.

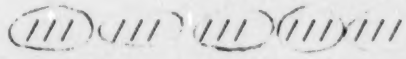
On page 59T of the teacher's guide it says:

Few children have any idea what happens when 2 numbers such as 27 and 63 are multiplied. This picture has been included to help children gain an appreciation of the magnitude or size of numbers. He goes on to say: There will inevitably be some children who experience difficulty with both the multiplication and division algorithms. It may be possible to give such children extra help by showing them how to manipulate concrete base 10 materials. (!) Again, the expectations are very unrealistic given the present conditions.

COMMENTS ON DIVISION (See appendix B)

The main criticism is aimed at the new approach for division in particular at the new and complicated algorithm and the constant mixing up of the grouping and sharing ideas.

Students at this stage have a good understanding of grouping. When you ask a grade 5 student to do 15 ÷ 3 he will do

 and answer: How many groups of 3 are there in 15?

Students know that this can be written as

$$\begin{array}{r} 5 \\ 3\overline{)\,15} \\ -15 \\ \hline 0 \end{array}$$

The question asked here is:

How many times can you subtract 3 from 15? or
How many losts of 3 can be subtracted from 15?
This algorithm can be easily extended to include more difficult problems, even decimals. When it comes to the decimal point, we should tell them to put it, and continue asking: How many times can you subtract ... etc. The point is TO PUT IT!

From my experience in teaching remedial arithmetic, we should stick to the old algorithm and not confuse teachers and students by trying to explain everything in detail. When you use calculators it is good enough to know how to press the keys in the proper order without having to know all the works.

My students, which include Post Vocational students as well as direct entry students, often tell me after having explained the traditional approach, that they for the first time have understood what they are doing. Of course, they don't understand, but they feel confident since they obtain the right answer.

After having explained the new division algorithm by using the sharing idea, and after students have drawn pictures to work out answers the sharing idea is abandoned temporarily. On page 69-70-71 harder examples are explained by using grouping.

When it comes to the division of decimals by whole numbers on pages 125-128-129 the sharing idea is re-introduced.
Finally on page 150 of the Grade 8A book (see appendix C)
the following advise is given:

We have shortened the first way to get the second way. In future, try the short way to do all your divisions.
This is the way you and I have always done it, and done it well.

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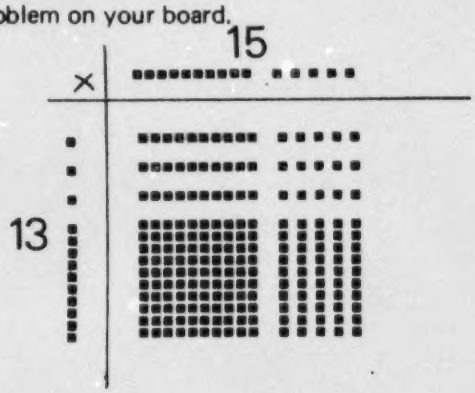
APPENDIX A

From:

Teachers and Students Text - Grade 6
Indigenous Mathematics Project
Working Paper 13
1980

READ THIS:

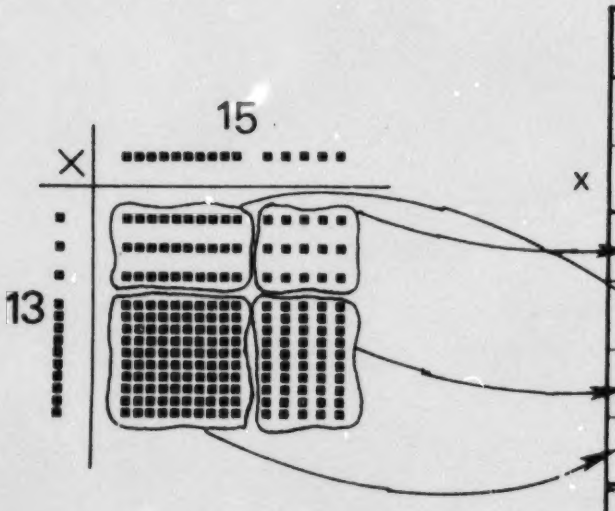
1. The picture below shows 13 x 15.
   It shows 13 lots of 15.
   To find the answer to 13 x 15 we count the units in the rectangle shape.
   Set up this problem on your board.



Things you need:

- base-10 materials
- board
- chalk
- squared paper
- pencil

2. We put boxes around the pictures of 3 x 5, 3 x 10, 10 x 5 and 10 x 10 to help us count the small squares (units).
   Altogether there are 15 + 30 + 50 + 100 = 195 small squares.
   We write the multiplication in a place-value table.

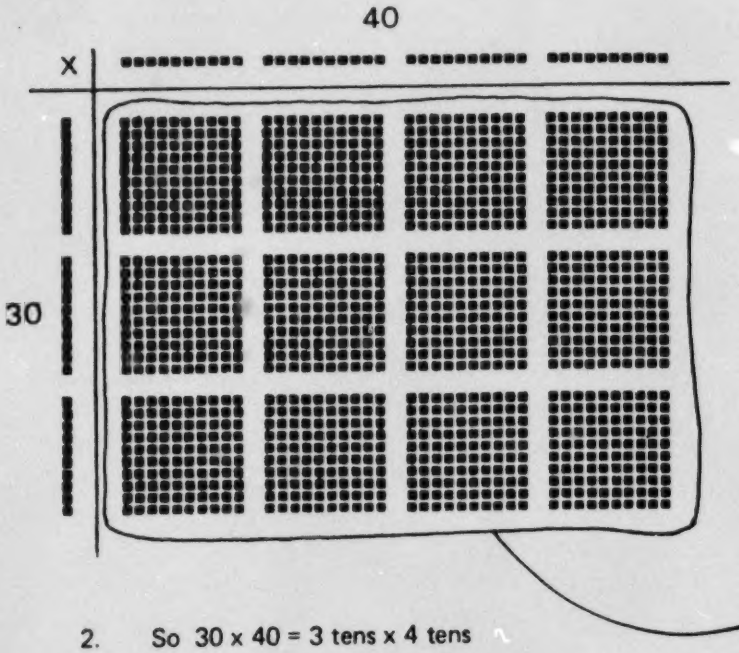


	100s	10s	1s	
		1	5	
×		1	3	
		1	5	← 3 lots of 5 ones
		3	0	← 3 lots of 1 ten
		5	0	← 10 lots of 5 ones
	1	0	0	← 10 lots of 1 ten
	1	9	5	← 13 lots of 15

3. So 13 lots of 15 = 13 x 15
   = 195

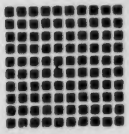
READ THIS:

1. Here is a picture that shows 30 x 40.



	1000s	100s	10s	1s
			4	0
X			3	0
				0
				0
			0	0
	1	2	0	0
	1	2	0	0

2. So 30 x 40 = 3 tens x 4 tens
   = 12 hundreds
   = 1 200

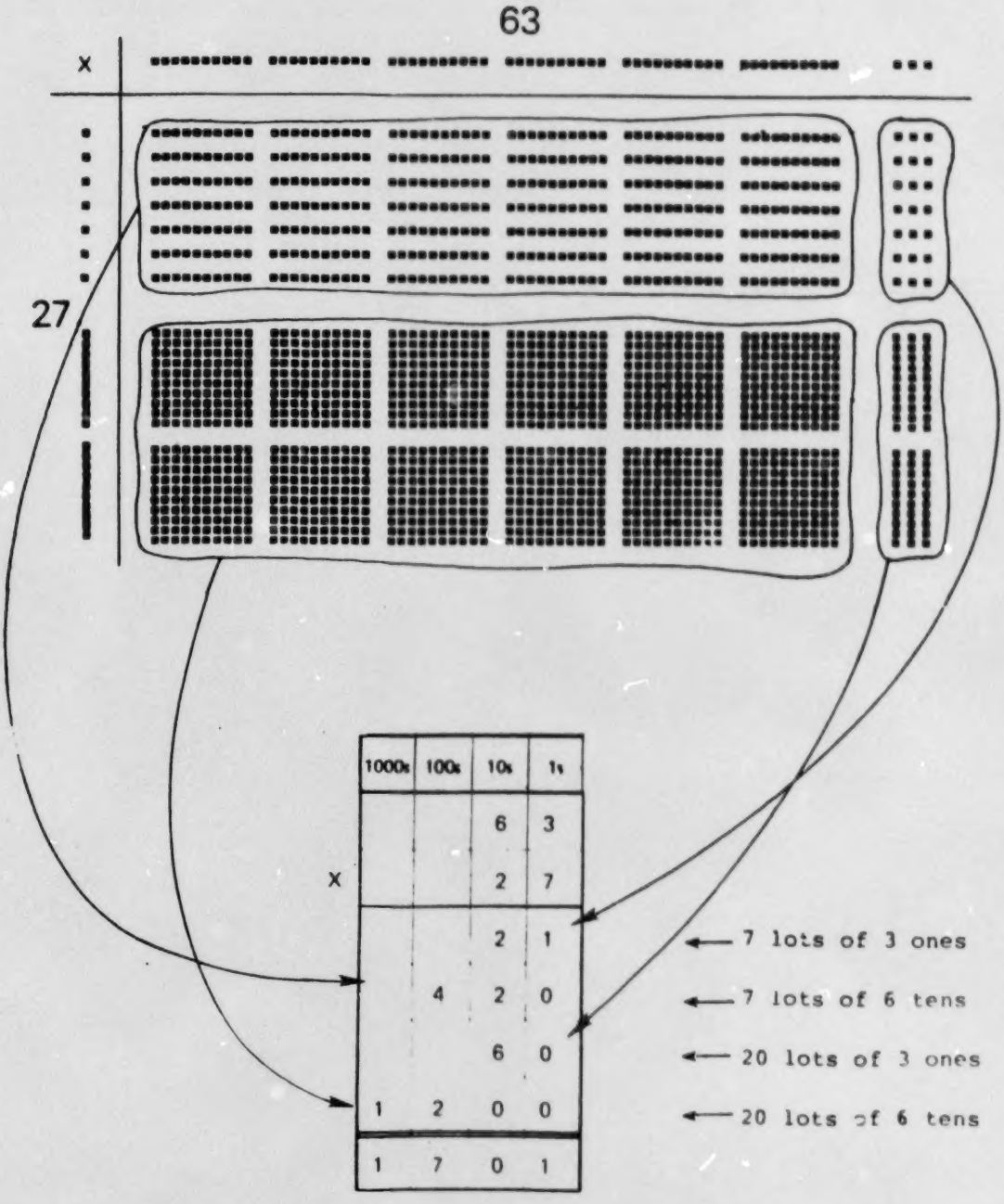
3. Use  to show each problem below on your board.

   Write the multiplication in a place-value table on squared paper.

   (a) 20 x 50
   (b) 40 x 60
   (c) 70 x 30
   (d) 90 x 70

READ THIS:

1. Look at this multiplication.



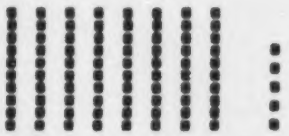
APPENDIX B

From:

Mathematics Our Way - 7A
Department of Education
Papua New Guinea, 1981.

READ THIS

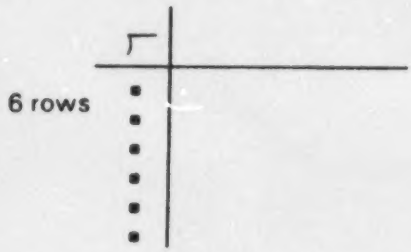
The divisions that we have done all work out evenly.
Lots of divisions don't work out evenly.
Here is a picture of 85 small squares.



Altogether there are 8 tens and 5 ones.

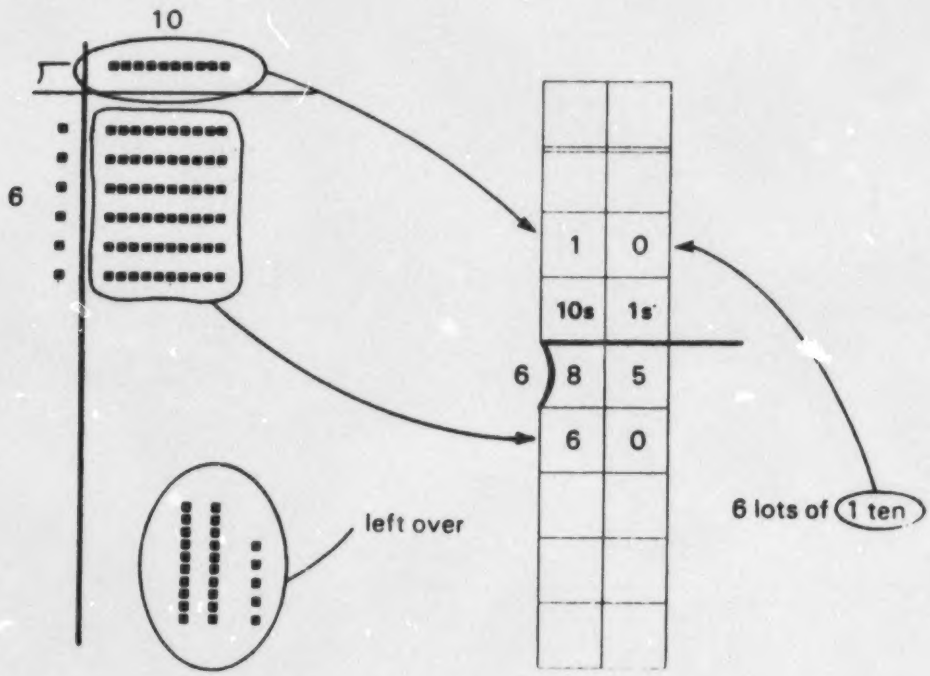
Suppose we want to do $6\overline{)85}$. We do it in steps.

Step 1 We set up a dividing table with 6 rows.



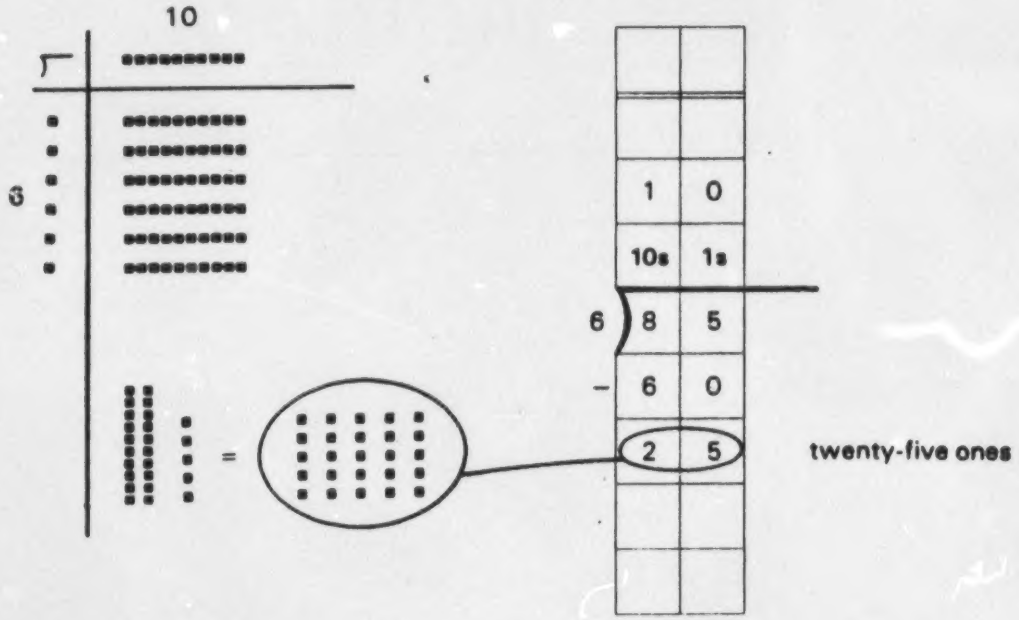
We use the table to share 8 tens and 5 ones equally among 6 rows.

Step 2 6 tens are shared equally among the 6 rows.
2 tens and 5 ones are left over.

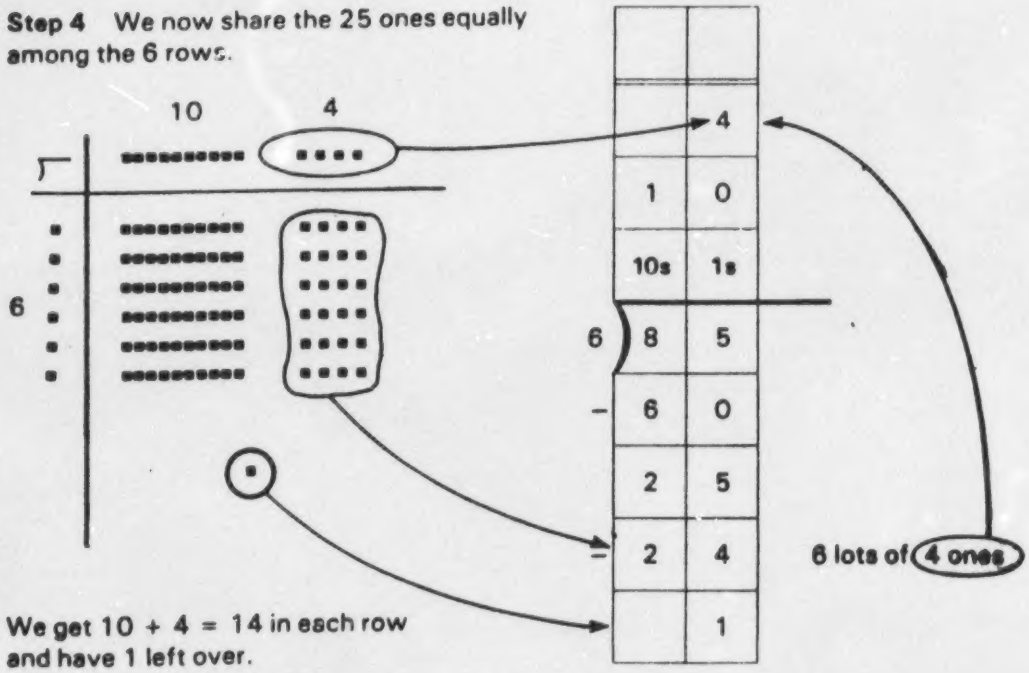


We show that we have put 1 ten in each row by putting 1 ten at the top where the answer goes.

Step 3 We have used 6 lots of ten and have 2 lots of ten left.
We change the 2 tens into 20 ones. So altogether we have 25 ones.

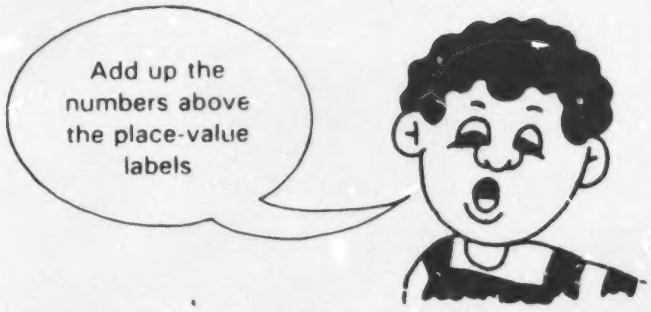


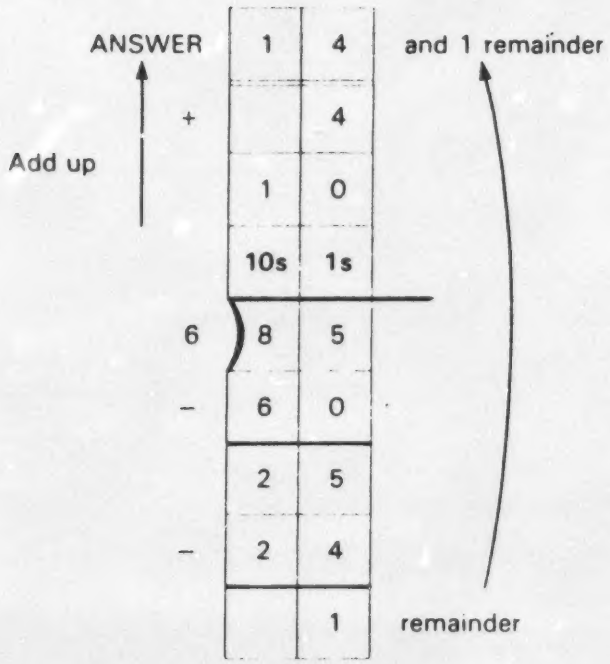
Step 4 We now share the 25 ones equally among the 6 rows.



We get 10 + 4 = 14 in each row
and have 1 left over.

Step 5 We add to get the answer.

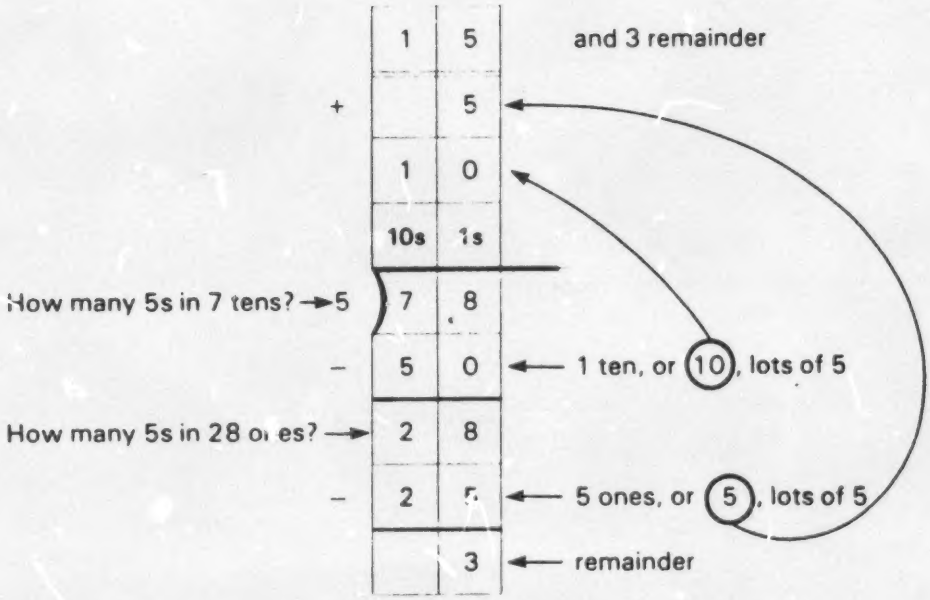




So 6)85 = 14 and 1 remainder.

Do this exercise (8)

1. Use pictures to work out the answer to each division below.
   Record in place-value tables what you do after each step.
   (a) $8\overline{)76}$
   (b) $7\overline{)83}$
   (c) $5\overline{)69}$
   (d) $9\overline{)53}$
2. In the next example, we have just done the working. We haven't used a dividing table.


1 5
and 3 remainder
+ 5
1 0
10s 1s
How many 5s in 7 tens? → 5) 7 8
– 5 0 ← 1 ten, or (10), lots of 5
How many 5s in 28 ones? → 2 8
– 2 5 ← 5 ones, or (5), lots of 5
3 ← remainder

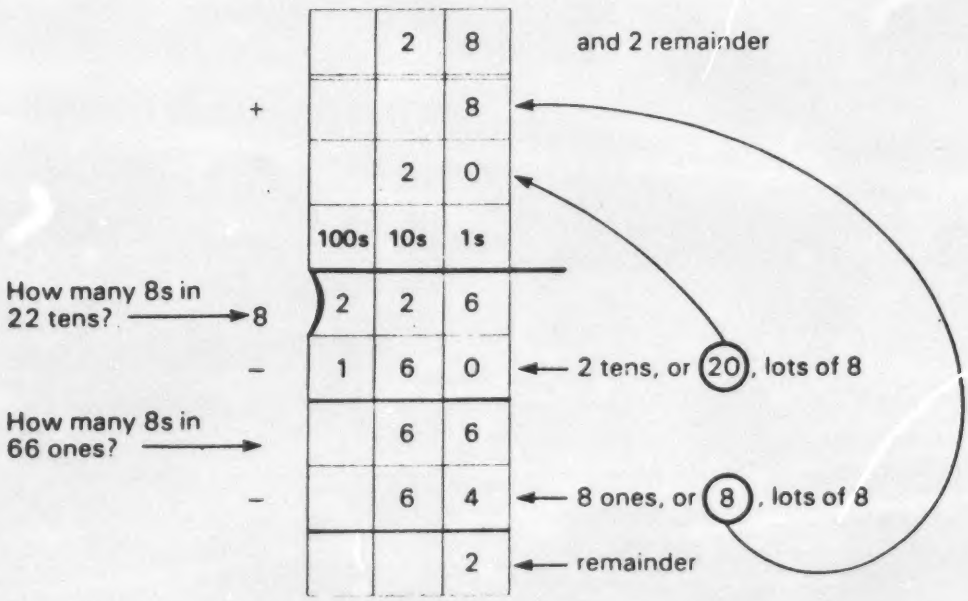
So $5\overline{)78}$ = 15 and 3 remainder.

Now work out each division below. Use squared paper.

(a) $6\overline{)96}$ (b) $3\overline{)48}$ (c) $7\overline{)49}$

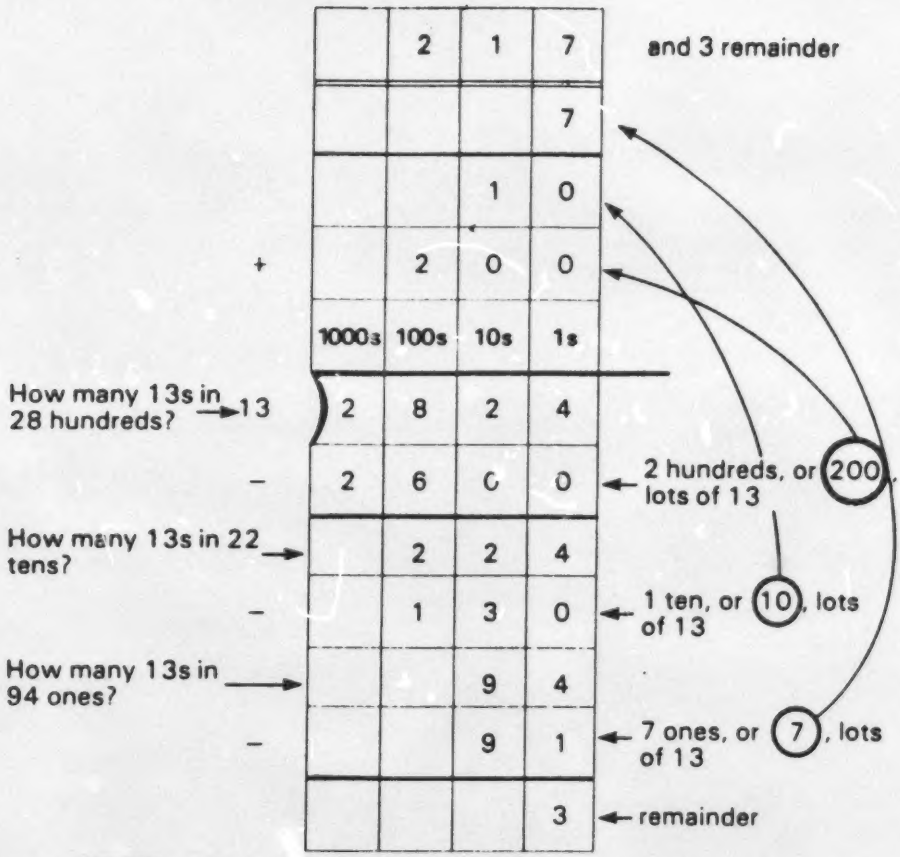
(d) $8\overline{)86}$ (e) $9\overline{)67}$ (f) $4\overline{)93}$

3. Here is a harder example. Look carefully at it.


2 8
and 2 remainder
+ 8
2 0
100s 10s 1s
How many 8s in 22 tens? → 8) 2 2 6
– 1 6 0 ← 2 tens, or (20), lots of 8
How many 8s in 66 ones? → 6 6
– 6 4 ← 8 ones, or (8), lots of 8
2 ← remainder

Example 2

Work out $13\overline{)2824}$


2 1 7
and 3 remainder
7
1 0
+ 2 0 0
1000s 100s 10s 1s
How many 13s in 28 hundreds? → 13
2 8 2 4
– 2 6 0 0
2 hundreds, or 200, lots of 13
How many 13s in 22 tens? →
2 2 4
– 1 3 0
1 ten, or 10, lots of 13
How many 13s in 94 ones? →
9 4
– 9 1
7 ones, or 7, lots of 13
3
remainder

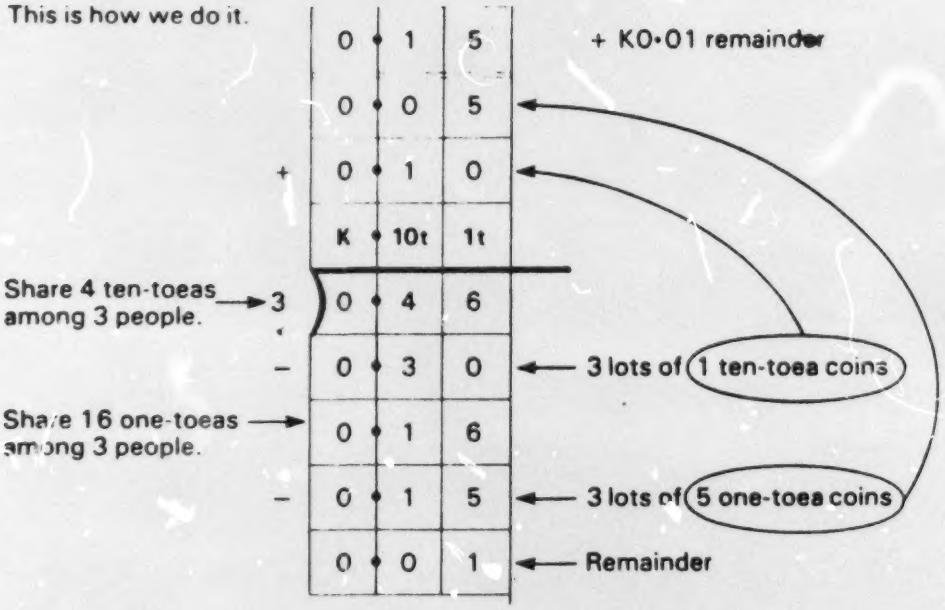
So $13\overline{)2824} = 217$ and 3 remainder.

Do this exercise (9)

1. (a) $12\overline{)263}$ (b) $12\overline{)407}$ (c) $14\overline{)292}$
   (d) $13\overline{)310}$ (e) $16\overline{)491}$ (f) $12\overline{)324}$
   (g) $18\overline{)263}$ (h) $19\overline{)361}$

2. You will often find a different sign to tell you to divide.
   We can write $6\overline{)36} = 36 \div 6$.
   But $36 \div 6$ still means "How many 6s in 36?"
   Divide.
   (a) $426 \div 20$ (b) $616 \div 23$ (c) $536 \div 26$
   (d) $842 \div 29$ (e) $396 \div 31$ (f) $629 \div 52$
   (g) $893 \div 37$ (h) $721 \div 39$

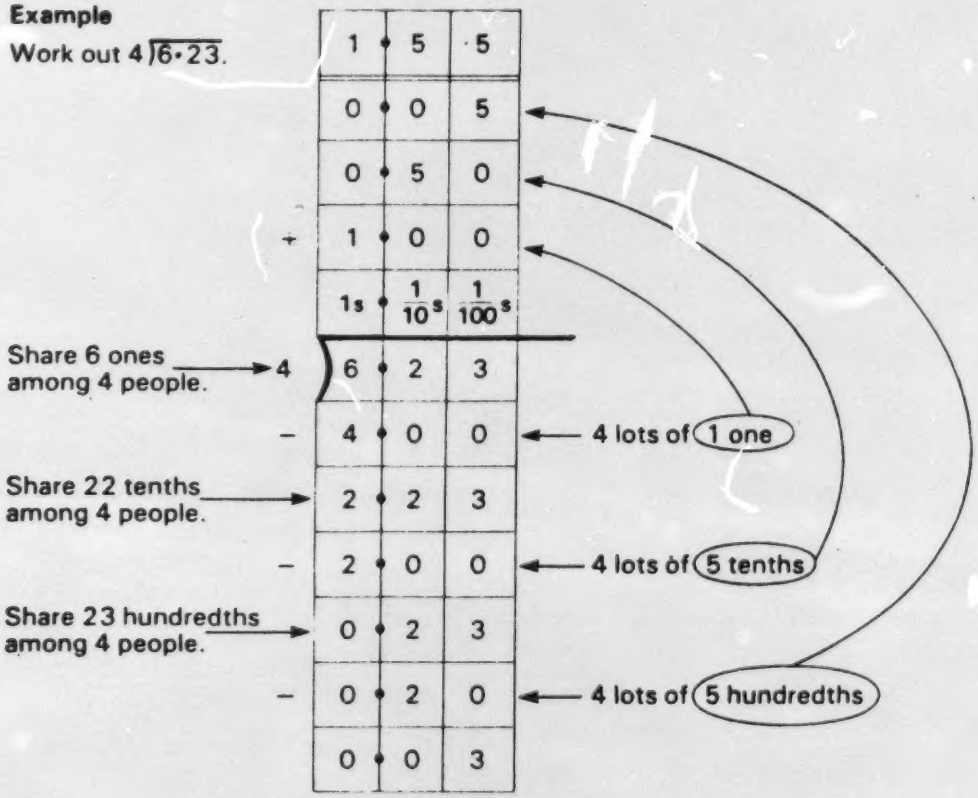
This is how we do it.


0 • 1 5
+ K0·01 remainder
0 • 0 5
+ 0 • 1 0
K • 10t 1t
Share 4 ten-toeas among 3 people.
3
0 • 4 6
– 0 • 3 0
3 lots of 1 ten-toea coins
Share 16 one-toeas among 3 people.
0 • 1 6
– 0 • 1 5
3 lots of 5 one-toea coins
0 • 0 1
Remainder

Each person gets K0·15; there is 1 toea left over. (We can't share 1 toea.)
We divide decimal numbers in the same way that we divide money.

Example

Work out $4\overline{)6{\cdot}23}$.


1 • 5 ·5
0 • 0 5
0 • 5 0
+ 1 • 0 0
1s • 1/10s 1/100s
Share 6 ones among 4 people.
4
6 • 2 3
– 4 • 0 0
4 lots of 1 one
Share 22 tenths among 4 people.
2 • 2 3
– 2 • 0 0
4 lots of 5 tenths
Share 23 hundredths among 4 people.
0 • 2 3
– 0 • 2 0
4 lots of 5 hundredths
0 • 0 3

So $4\overline{)6{\cdot}23}$ = 1·55 and 0·03 remainder.

Because the remainder (0·03) is very small, we don't write it in the answer. We say $4\overline{)6{\cdot}23}$ = 1·55 to 2 decimal places.

Do this exercise (8)

1. Do the working on squared paper.
   (a) Share K4·27 between 2 boys.
   (b) Share K6·19 among 4 boys.
   (c) Share K8·72 among 3 girls.
   (d) Share K7·84 among 5 girls.

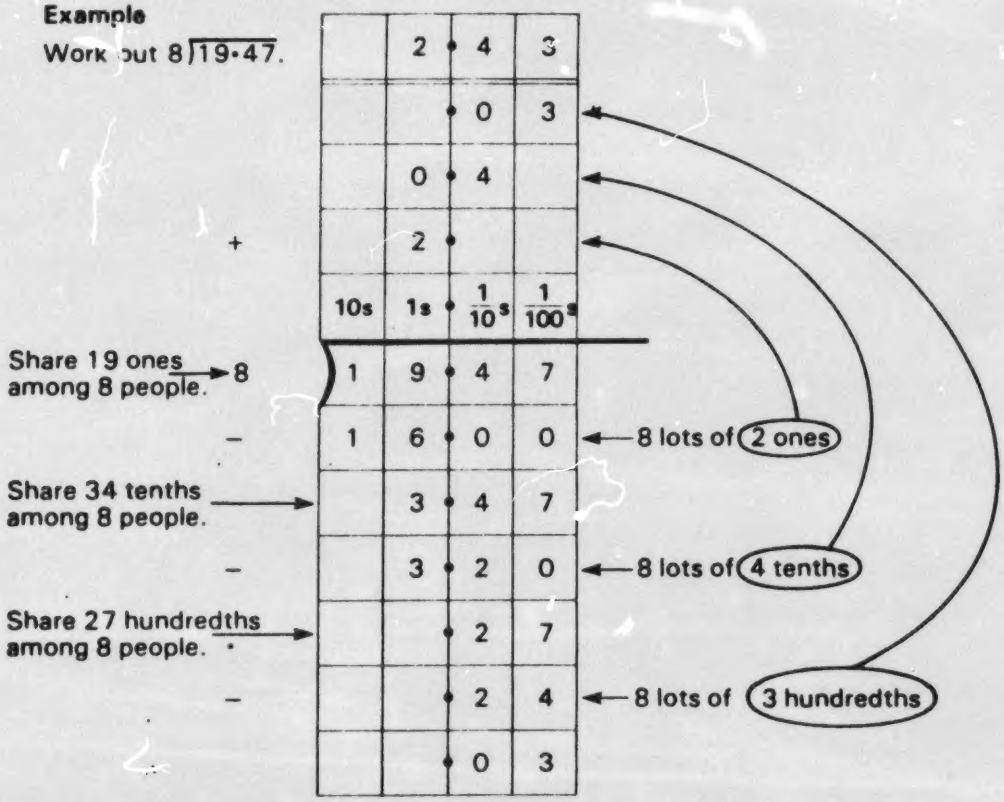
2. Divide on squared paper.
   (a) $4\overline{)8{\cdot}52}$ (b) $3\overline{)7{\cdot}18}$ (c) $2\overline{)5{\cdot}91}$ (d) $5\overline{)6{\cdot}07}$ (e) $3\overline{)8{\cdot}18}$
   (f) $7\overline{)9{\cdot}23}$ (g) $6\overline{)6{\cdot}48}$ (h) $3\overline{)8{\cdot}31}$ (i) $3\overline{)0{\cdot}62}$ (j) $4\overline{)0{\cdot}43}$

3. Divide on squared paper.
   (a) $2\overline{)1{\cdot}62}$ (b) $4\overline{)3{\cdot}17}$ (c) $8\overline{)6{\cdot}29}$ (d) $7\overline{)4{\cdot}63}$ (e) $6\overline{)2{\cdot}34}$
   (f) $4\overline{)1{\cdot}83}$ (g) $5\overline{)3{\cdot}07}$ (h) $2\overline{)1{\cdot}49}$ (i) $5\overline{)0{\cdot}33}$ (j) $9\overline{)0{\cdot}77}$

4. Here is a harder example. Look carefully at the working:

Example

Work out $8\overline{)19{\cdot}47}$.

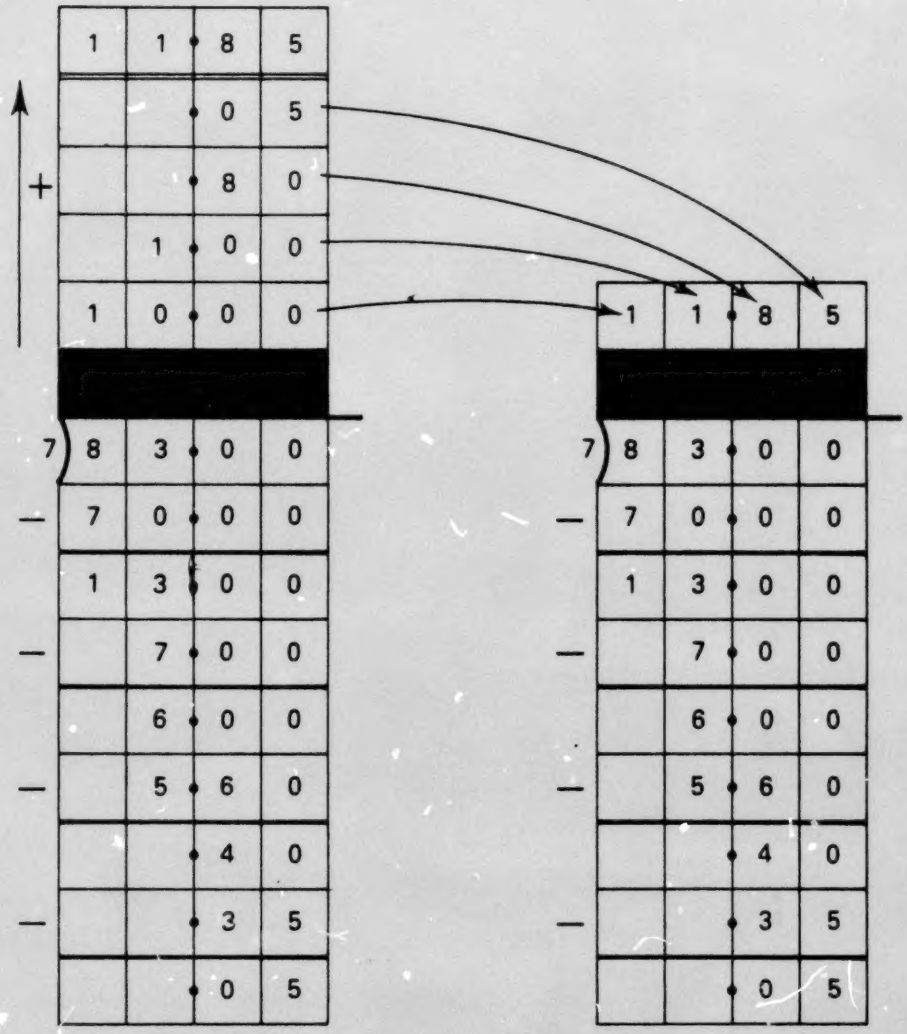

2 • 4 3
• 0 3
0 • 4
+ 2 •
10s 1s • 1/10s 1/100s
Share 19 ones among 8 people. → 8
1 9 • 4 7
– 1 6 • 0 0 ← 8 lots of (2 ones)
Share 34 tenths among 8 people. →
3 • 4 7
– 3 • 2 0 ← 8 lots of (4 tenths)
Share 27 hundredths among 8 people. →
• 2 7
– • 2 4 ← 8 lots of (3 hundredths)
• 0 3

So $8\overline{)19{\cdot}47}$ = 2·43 to 2 decimal places.

APPENDIX C

From:

Mathematics Our Way - 8A
Department of Education
Papua New Guinea, 1981.



We have shortened the first way to get the second way.

In future, try to use the short way to do **all** your divisions.

Do this exercise (4)

On squared paper, work out the answer to each division below.
(Use the short way.)

1. $6\overline{)47}$
2. $7\overline{)51{\cdot}3}$
3. $8\overline{)75{\cdot}6}$
4. $9\overline{)95{\cdot}8}$
5. $11\overline{)63{\cdot}2}$
6. $12\overline{)97{\cdot}2}$
7. $13\overline{)146{\cdot}2}$
8. $14\overline{)213{\cdot}6}$

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COMPUTER - ASSISTED INSTRUCTION AT UNITECH

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The title is probably rather misleading. Several courses at Unitech teach Computer Programming; more teach students how to use "utility" programmes or packages of programmes that have already been written for particular purposes (surveying, text storage and editing, and particularly statistical analysis of data). But there is very little use of what I think is properly described as computer-assisted instruction: that is, the use of programmes which interact with the student to introduce new material, question the student, analyse his or her response, and decide on the basis of that response whether to proceed, give further explanations or exercises, or (as a last resort) call in a live tutor!

The programmes that I have been using do not really fit into this category, but they probably come as close to it as any at Unitech. Most of them belong to two packages entitled "Algebra Drill and Practise" bought by the Computer Centre, published by Conduit, Iowa, USA. These cover a range of topics from directed numbers and collecting like terms to simple "word problems" (only of one type in the one programme, but still useful) and solutions of sets of two and three simultaneous linear equations.

Two programmes in use at the moment I have written; these are "TABLES" which is a very simple speed drill on number facts, and "DECIMALS". Each of these presents sets of ten questions of a selected type. The numbers in the questions are generated at random within a specified range, so it is quite possible to get repetitions, or a succession of very easy questions like "multiply by one" or "add zero". Programme DECIMALS, unlike TABLES, allows a second attempt if the first is wrong; if the second also is wrong it presents the correct answer, then waits to be restarted before giving the next question. It also differs in allowing a choice whether the user is told the time taken in answering the ten questions. A user who finds a particular type of question difficult can take it more slowly and carefully without worrying about the timing.

But the best way to describe them is to give you a practise run.

In the following pages, all except the lines of comments separated by rows of dots is printout from the computer when these programmes were being run. So that you can tell what was printed by the computer and what was typed at the keyboard by the user, the user input has been underlined. This has been done by hand, so is probably the least reliable part of the text!

run cai:tables

Welcome to programme 'TABLES'.

This programme is intended to help you improve the speed and accuracy of your arithmetic tables, by giving you sets of ten questions and timing how long it takes you to answer.

If you want to stop the programme at any time (except in the middle of a set of ten questions), then type /S and press 'RETURN'.

Please type your name, and press 'RETURN': Alan

Hello, Alan. Have I got your name right?
Please type Y for yes or N for no, then press 'RETURN': y

Which operation would you like to practise?

1	Addition	: +
2	Subtraction	: -
3	Multiplication	: x
4	Division	: /

Your choice : sub

OK, Alan, here is a set of ten subtraction questions.
If you return a wrong answer, do not worry about it or try to fix it; the computer will go straight on to the next question.

Type R when you are ready to begin.
(If you do not want to do these questions, then type /S instead.)

r

77 - 3 = 74
correct.
91 - 1 = 90
correct.
87 - 4 = 73
wrong.
70 - 6 = 64
correct.
74 - 1 = 73
correct.
56 - 2 = 544
wrong.
31 - 2 = 29
correct.
81 - 9 = 72
correct.
94 - 8 = 82
wrong.
78 - 2 = 76
correct.

You scored 7 out of 10 in 44 seconds.
Would you like another set of subtraction questions?
Y or N : y

OK, Alan, here is a set of ten subtraction questions.
If you return a wrong answer, do not worry about it or try to fix it; the computer will go straight on to the next question.

. . . . . .

and so on as before. An improved score gets the response:

. . . . . .

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```
      You scored   9  out of  10 in   39  seconds.
      That's better.  Keep up the improvement!
      Would you like another set of subtraction questions?
                  Y  or  N : y
   .  .  .  .  .  .  .  .
     and a further improvement still:
   .  .  .  .  .  .  .  .
      You scored  10  out of  10 in   42  seconds.
      Perfect score.  Well done,  Alan!  Can you do it faster?
      Would you like another set of subtraction questions?
                  Y  or  N : y
   .  .  .  .  .  .  .  .
     until:
   .  .  .  .  .  .  .  .
      You scored  10  out of  10 in   23  seconds.
      This is your fastest perfect set so far on this run.
      Congratulations,  Alan!
      Would you like another set of subtraction questions?
                  Y  or  N : n

      Your best score on subtraction questions was  10 out of  10.
Your fastest time for this score was   23 seconds.

      Would you like to practise some more questions,  Alan?
      Y  or  N : n

      So long,  Alan.

      Come back and practise your TABLES again some other time.
   .  .  .  .  .  .  .  .
```

In one respect TABLES is more flexible than DECIMA: when it presents its menu of question types, it will give a set of (for example) subtraction questions if the first non-blank character in the user's response is any one of '2', 'S', 's' or '-'. Decima insists on having a numerical character to read. Note how fussy the operating system is about correct spelling:

```
   .  .  .  .  .  .  .  .

run cai:decema
?Can't find file or account

Ready
run cai:decima

      Welcome to programme 'DECIMALS'.

      This programme is intended to help you improve the speed and accuracy
of your simple work with decimals.  These questions are all easy enough
to be done in your head.
      You choose which type of question you want to practise;
the computer will then give you sets of ten questions of this type.
If you wish, it can tell you how long you took to answer each set.


      If you want to stop the programme at any time (except in the middle
of a set of ten questions), then type  /S  and press 'RETURN'.
      Please type your name,  and press 'RETURN': Bety

      Hello,  Bety.  Have I got your name right?
Please type  Y  for yes  or  N  for no,  then press 'RETURN'.
                  Y  or  N : n
      Then let's try it again.
```

```
      Please type your name,  and press 'RETURN': Betty

      Hello,  Betty.   Have I got your name right?
Please type   Y   for yes  or   N   for no,  then press 'RETURN'.
                  Y  or  N : y

      Which operation would you like to practise?

1.    Add a one-digit number in the last place-value.          3.7 + 0.8
2.    Add a one-digit number in any place-value.               0.65 + 0.7
3.    Subtract one digit from the last place-value.            950 - 70
4.    Subtract one digit from any place-value.                 0.57 - 0.2
5.    Subtract from a round number - 'giving change'.          1.00 - 0.38
6.    Multiply by a power of 10.                               3.25 x 0.01
7.    Multiply,  using tables and place-value.                 0.7 x 0.06
8.    Divide,  involving a power of 10.                        3.25 / 32.5
9.    Divide,  using tables and place-value.                   560 / 0.8
0.    Divide up to six decimal places (frac->decimal).         3 / 7

      Your choice  : Add
      Please answer with one of the digits  1 to 9  or 0.
      Which operation would you like to practise?
  .   .   .   .   .   .   .
    And it gives the whole list again.  This time:
  .   .   .   .   .   .   .
      Your choice  : 2

      Would you like to be told your time for each set of ten questions?
                  Y  or  N : y

      OK,  Betty, here is a set of ten questions of type  2.

      Please type  R  when you are ready to go on.
      (You can skip this set of questions by typing  /S  instead.)
                  r

            0.24  + 0.08 = 0.32
                                          correct.
            0.12  +  2 = 0.14
      Wrong;  think carefully about it and try again.

            0.12  +  2 = 2.12
                                          Yes,  that's it.
           45000  + 70000 = 115000
                                          correct.
            0.21  + 0.00 = .21
                                          correct.
              18  + 0.1 = .19
      Wrong;  think carefully about it and try again.

              18  + 0.1 = 1.9
                                          No,  still wrong.
      The correct answer is:
              18  + 0.1 = 18.1

      Please type  R  when you are ready to go on.
      (You can skip this set of questions by typing  /S  instead.)
                  r
           0.007  + 0.02 = .027
                                          correct.
  .   .   .   .   .   .   .   .
Let's skip to another user and a different type:
  .   .   .   .   .   .   .   .
```

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Your choice : 5

Would you like to be told your time for each set of ten questions?
Y or N : y

OK, Charlie, here is a set of ten questions of type 5.

Please type R when you are ready to go on.
(You can skip this set of questions by typing /S instead.)
r

50.0 - 36.1 = 13.9
correct.

2.00 - 1.85 = .15
correct.

2.00 - 0.82 = .18
Wrong; think carefully about it and try again.

2.00 - 0.82 = 1.18
Yes, that's it.

10000 - 2180 = 7820
correct.

100.0 - 92.6 = 7.4
correct.

10.00 - 6.65 = 3.35
correct.

0.500 - 0.383 = 0.117
correct.

50000 - 46800 = 320
Wrong; think carefully about it and try again.

50000 - 46800 = 420
No, still wrong.

The correct answer is:
50000 - 46800 = 3200

Please type R when you are ready to go on.
(You can skip this set of questions by typing /S instead.)
/s

You got 6 correct at the first attempt,
and 1 correct at the second attempt, out of 8.

It took you 109 seconds to give your answers.
Would you like another set of questions of type 5?
Y or N : n

Your best score on questions of type 5 was:
6 correct at the first attempt,
and 1 correct at the second attempt, out of 8.
Your fastest time for this score was 109 seconds.

. . . . . .

Some quick samples: type 6:

. . . . . .

0.001 x 28 = .028
correct.

7.56 x 0.01 = .0756
correct.

0.01 x 9540 = 95.4
correct.

10 x217000 = 2170000
correct.

1 x 38700 = 38700
correct.

```
OK,  Di, here is a set of ten questions of type  7.
. . . . . .
        0.2  x 0.010 = .002
                                        correct.
        700  x  90 = 63000
                                        correct.
      0.005  x 0.011 = .00055
Wrong;  think carefully about it and try again.

      0.005  x 0.011 = .000055
                                        Yes,  that's it.

         12  x  300 = 3600
                                        correct.
       5000  x 0.012 = 60
                                        correct.
. . . . . .
My students find type  8 surprisingly difficult:
. . . . . .
      494000  /  100 = 4940
                                        correct.
      553000  /  5530 = 100
                                        correct.
         7.7  /  0.01 = .077
Wrong;  think carefully about it and try again.

         7.7  /  0.01 = 770
                                        Yes,  that's it.

        4.90  /   4.9 = 10
Wrong;  think carefully about it and try again.

        4.90  /   4.9 = 1
                                        Yes,  that's it.

        5.54  /  5540 = .001
                                        correct.
        30.9  / 0.309 = 100
                                        correct.
. . . . . .
Your choice  : 9
. . . . . .
         2.4  /  2 = 1.2
                                        correct.
      0.0001  / 0.05 = .02
Wrong;  think carefully about it and try again.

      0.0001  / 0.05 = .002
                                        Yes,  that's it.

        0.01  / 0.02 = .5
                                        correct.
        0.96  / 0.8 = 1.2
                                        correct.
      250000  /  5000 = 50
                                        correct.
. . . . . .
```

That should be enough to give you the idea. The big advantage of doing this sort of thing with a computer is the instant and reliable feedback on each answer. The feedback comments after sets of ten should also encourage a student to improve by competing against himself. This programme doesn't teach concepts or methods of working with decimals; but for a student who has these foundations and needs practise, it gives structure and incentive for that practise.

THE DEVELOPMENT OF TIME CONCEPTS AND THE USE OF COMPARATIVES AMONG PAPUA NEW GUINEANS

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Much has been written about the inability of Papua New Guinean students to correctly interpret time concepts. This paper discusses aspects of students' abilities at time tasks and looks at some of the generalizations which have been made by previous commentators. Many of the previous assertions are questioned and a different approach to teaching such concepts is suggested.

INTRODUCTION

Technical and professional fields require a good understanding of time concepts, not only for the many organisational aspects in which time is involved but also as a prerequisite to understanding many of the scientific concepts which their professional field requires.

Some of the everyday situations which require interpretation of time are:

language - the tenses of verbs and comparative terms such as before and after, sooner and later, etc.
history - the placing of past events in relation to each other,
facts - the days in a week, minutes in an hour, etc.
estimation- the length of 30 seconds, 2 hours etc.
clocks - both reading and interpreting
calendars -
ages - of people, equipment
timetables- airline schedules.

Time is critical in many professional areas. A nurse, for example, is required to calculate and record dosage schedules in hours, pulse and respiration rates per minute, elapsed time since a previous observation, time differences between observations, patient's ages given only their birth date, intra-veneous drip rates both in drips per minute and in millilitres per hour, the dates of confinement or length of term if given the date of the last menstrual period, and well as being able to estimate the times associated with each of these tasks.

It is clearly an important objective of nursing education courses to ensure that the students are competent at the time tasks which they will meet. The same is true for many professional and technical courses.

It is also true that many Papua New Guinean students perform poorly on some time related tasks and in fact, to westerners, it appears that Papua New Guineans generally have quite a different approach to time. This paper presents some results of written questions given to Provincial High School students, discusses some of the conclusions of previous time studies and suggests a different interpretation which can be placed on their results, as well as noting some implications that this has for the classroom teacher.

WRITTEN TEST RESULTS

A written test on time was administered to a group of 77 grade 7 students and 72 grade 9 students from Provincial High Schools in Lae. The sample was predominantly boys and a considerable number would be living in an urban setting. It can be assumed that their exposure to western time concepts would be greater than that of average Papua New Guinean students of their age. Table 1. shows facilities in per cent on some of the easier questions.

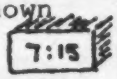
	GRADE 7	GRADE 9
Draw the hands on this clockface (given) to show the time shown on this digital clock. 	71	97
What time is shown?	71	95
What time is shown?	50	85
What is the time now?	65	85
What day of the week is May 15?	84	92

Table 1: Facilities on some easier time questions (%).

These question refer to very basic time tasks, and while the grade 7 results are not ideal they do indicate that a majority of the students can perform these tasks. The grade 9 results are quite good and indicate that nearly all students are able to perform such tasks. Table 2 shows the facilities of other questions on the same test.

	Grade 7	Grade 9
What is the date 9 months and 7 days later than June 25, 1980?	0	10
What is the date 5 days before September 1.	7	50
A baby is born on 15 May. On what date will he be 2 months old? (A calendar was supplied for these questions.)	7	30
If the time is 8 o'clock in the morning, what was the time 10 hours ago?	18	60
On the clockface (given) draw where the hands are 40 mins later than the time shown.	13	57

Table 2. Facilities on more difficult time questions(%).

The responses from the grade 7 students are very poor, and the grade 9 students are also experiencing considerable difficulty with these questions. On first impression it is reasonable to conclude that the students have a poor understanding of these time situations. The questions are a little more complicated than those in Table 1, but it is not clear why they prove to be so much more difficult. If the factors which cause these difficulties can be identified, then it will be easier to develop more meaningful teaching strategies.

PREVIOUS STUDIES

There have been a small number of studies of time concepts from a mathematical viewpoint in Papua New Guinea. Lewis and Mulford (1974) posed a variety of piagetian style time conservation tasks to secondary level students. They found that 28% of their sample did not conserve time and that 67% of the students were unable to estimate time durations. They conclude that this has serious implications for teaching programs. Meek (1978) in a review of socio-linguistic studies which relate to mathematics notes a marked inability of students to master time-related tasks and says that this is due to a lack of words in traditional languages which describe time periods. He mentions difficulties with the tenses of verbs as a contributing factor and refers to the absence of comparatives.

Prince (1970) reports a study among students at the University of Papua New Guinea which investigated a number of time tasks such as estimation, ordinality, perceptions of time, the scientific basis of time, and simultaneity. He says that student difficulties are mainly linguistic, due to misuse of tenses, and historical due to the lack of a recorded history. He indicates that it is the absence of these in traditional cultures which is the basis of the student difficulties with time tasks and he concludes by saying

> " It is unlikely that New Guineans can be taught ideas of time, except as the course of teaching gives them time to learn by experience."

There is an underlying theme in the discussions on time concepts that Papua New Guineans are experiencing great difficulty with understanding time in a western context, and that the difficulties are related to the lack of similar concepts in their traditional cultures.

Selden and Sullivan (1982) report findings from two field studies which contradict many of the above conclusions. They report that some cultures (at least) have quite clear systems for marking and identifying time periods, both short and long term. They indicate the existence of quite definite tenses in the verbs and the well developed historical sense among village people, at least within living memory. There appears to be clear and detailed sets of concepts and language to deal with time situations which occur in their lives. If these two narrow investigations are in any way representative of the situation generally it may mean that the teaching approaches suggested by earlier commentators are not appropriate, and that teachers could do well to base their teaching of western time ideas on the traditional concepts which exist in the students' culture.

COMPARATIVES

Meek (1978) mentioned the absence of comparatives in Papua New Guinean languages. This was also stressed by linguists at the Summer Institute of Linguistics in discussions with the authors. Comparatives such as more and less, before and after, etc. are fundamental concepts which are used frequently in the primary mathematics classroom and which are essential for any secondary mathematics studies. Of course, the use of "more and less" to compare objects is quite different to the use of "before and after" to compare events, timewise. They are treated together here, because the difficulties in translation appear to be similar, and the comments being made here apply to both types.

The following are some examples of the way that one Papua New Guinean language, Gawal from the Morobe Province, translates sentences involving comparatives:

Who was born before World War 11?

Asa dinda koc ing mung su goc sing meng?

Who mother gave birth him before and then war came

Who was born after World War 11?

Asa sing meng su goc ing dinda koc ing?

Who war came before and then him mother gave birth

I have more coconuts than you.

Au neng ndip hoc gelec aom nem

I my coconut more than you

To translate a sentence like "We went to the garden, and before that we went fishing", it is said as "we fished first and then we went to the garden". It appears that there are some comparatives, at least, but that they only indicate one direction of comparison. This is not to say that this system of comparison is ambiguous or unclear; in fact it appears to be a complete and consistent system. Nevertheless, the student must learn in English which uses comparatives in more complex ways than does the Gawal language, for example. It can not be assumed that the student will make the transfer to English easily, nor even that the student completely understands his own system.

Jones (1981) discusses the uses of comparitives with Community School children. He reports results of a study which investigated pupil responses to questions such as:

What is more, 3 or 5?
What is 3 more than 5?
The number 6 is 2 more than what number?

There are three different uses of the word "more" in these tasks. At grade 2 level few students differentiated between the usage and even at grade 6, fewer than half of the students could appreciate the difference. The results that he presents indicate clearly that the intricacies of the language used in these questions are not understood by the students at the end of community school. The questions presented in Table 2. above also use comparatives in a similar way. It may be that the difficulties experienced by the students on these questions may be more due to difficulties with language than to a lack of ability at interpreting time situations.

The key questions used in Lewis and Mumford were heavily loaded with comparatives in the same way. Prince (1970) also based his study on questions which were dependent on an understanding of comparatives in particular, and his conclusions about abilities at time tasks must be viewed in this context.

Table 3 presents results on some questions which were common to both the Jones (1981) study and to the test mentioned earlier.

QUESTION	Grade 4	Grade 6	Grade 7	Grade 9
It is now 5 o'clock. What was the time 3 hours ago?	6	45	47	77
A trade store is open from 1.00 p.m. to 4.00 p.m How long is it open?	6	17	34	67
It is now 2.00, what will the time be in 2 hours time?	18	33	88	100
The number 8 is 2 more than what number.	15	29	65	96

Table 3. Between grade comparison on time tasks.(%)

These results indicate , as Jones suggests, that there is quite a marked improvement in performance on these tasks over the grades. It may be that the improvement is as much due to the development of the necessary language skills to interpret the comparatives and other time words as it is to increased comprehension of time situations. It is also true that the development seems to occur at a later stage than one would consider necessary if these concepts are to be used in a meaningful way by the mathematics teachers.

It is clear in the case of comparatives that the English usage is different to the usage in traditional languages. While the traditional languages only compare things in one way, English can use many different ways. If the differences between the language systems are never pointed out to the students, then they will not even be aware of any misconceptions that they might have about the use of comparatives in English.

DISCUSSION

The point argued above is that there are many time concepts extant in Papua New Guinean cultures and there are even clear and unambiguous ways of comparing objects and situations. It is however unfortunately true that many Papua New Guinean students do not learn these concepts in their own language and context, since they are sent to school at a time when they would normally learn such ideas at home.

Western pre-schools and junior primary classes devote considerable time and effort to teaching the use of comparatives in various contexts. The teaching is based on the language skills which the students already possess. Later teaching of arithmetic processes and the development of number and time skills are greatly dependent on the understanding of these prior concepts.

In contrast, Papua New Guinean students must first learn the language of instruction and at the same time they are expected to learn the concepts which are being taught at a corresponding level in western schools.

It is not surprising then that Papua New Guinean students do not master time tasks and the use of comparatives until later than their western counterparts. This slower development obviously will have a significant impact on the development of many higher order mathematical skills which are dependent on an understanding of these concepts. It could well be that teachers are forced to resort to rote methods of instruction simply because the students have not developed sufficient understanding of these preliminary ideas. It may also explain the Papua New Guinean phenomenon of the "mute" student whereby many students sit silently in class and seldom contribute to class discussions. This could be a product of the early years where they were unaware of what was happening.

Teachers of migrants in Australia teach many concepts in English by referring to the concepts in the students' first language, and the students can then transfer the information from their first language to English. It may be that this method could be used to advantage in Papua New Guinea. If students are first taught about their own time systems and methods of comparison, then when the concepts are taught in English, at least the students will have something which they can understand as the basis for their learning.

SUMMARY

Papua New Guinean students do perform poorly on time tasks. This may be as much due to difficulties with language as it is with time.

Time concepts and methods of comparison seem to be more developed in traditional society than has been previously been recognised.

Papua New Guinean students should be first taught to understand their own time systems and comparatives, and then this should be used as a basis for instruction on the western systems.

Teachers and researchers should recognise the difficulties which many students experience with language in general, and comparatives in particular, and not draw hasty conclusions about the abilities of students in particular contexts.

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REGIONAL COOPERATION: PEACESAT, PRIME, AND THE SPMLC

HELEN WHIPPY

MATHEMATICS DEPARTMENT

PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

INTRODUCTION

There is a great deal going on in Mathematics Education in Papua New Guinea, a lot of it unpublished and unrecognised. Indeed, some workers do not even realize that what they are doing is research or that it is "good enough" to tell people about.

This paper gives a brief account of three avenues through which mathematics educators in Papua New Guinea can communicate with their counterparts throughout the Pacific. Any exchange of information - through journals, seminars, or competitions, will encourage all participants to the benefits of mathematics teaching and learning throughout the Pacific.

PEACESAT

The Pan-Pacific Education and Communication Experiment by Satellite (PEACESAT) began in 1971/2 with terminals in Suva Fiji, Tonga, Honolulu, and Wellington, New Zealand. A look at Figure 1, will confirm that there are 17 terminals in operation now.

PEACESAT is a demonstration project, using an old NASA satellite, ATS-1, that was supposed to have ended its useful life in 1973. It's objective is to "study the benefits arising from direct conference communications between groups with common interests in widely separated countries of the Pacific". (Satellite, no. 179, 1982). Almost anyone may use it, as long as it is not for commercial purposes. The network is to be used by persons and institutions engaged in research, education, health, and other community services. The advantage is the participants have direct voice-to-voice communications available at no cost. Most exchanges are fairly informal. The satellite demonstration will show you how to go about participating in or initiating a session.

This is the most immediate and direct way Papua New Guinea has of participating in Regional discussions. The Papua New Guinea government has often found it useful - for example, for organizing the South Pacific Festival of Arts in Port Moresby in 1980.

In Mathematics Education, for the past two years a number of members of the Mathematics Education Centre and Mathematics Department have participated in, and increasingly often, chaired a series of seminars of Regional interest to mathematics educators. Usually, it was not the discussion itself that was found most useful, but the follow-up contacts. It is a way of finding out about what's going on outside of Papua New Guinea, but in the Pacific Region and one means of making contact with colleagues in the Pacific.

PRIME MATHEMATICS JOURNAL

Tracing the history of PRIME was like solving a mystery. It began in 1969 in Papua New Guinea as Polygon, a mathematics newsletter/magazine published by the Papua New Guinea Department of Education Curriculum Unit. It was aimed at Mathematics teachers - at Community School to grade 10 level. In 1973, presumably with a change of personnel, it was superseded by Mathsnews, again, as far as can be guessed looking back, aimed at the same group, published by the Mathematics Education Centre. At about the same time, the Mathematics Department at the University of the South Pacific in Suva, Fiji began a fairly substancial journal/magazine called Maths Forum. It tried to reach the same group, teachers of mathematics in Forms 1-4, throughout the Pacific Region, with games, teaching tips and news of Mathematics Associations. Eventually in about 1975/6 with a shift-around of personnel in the Region, Mathsnews and Maths Forum were combined to give us a truely regional journal, PRIME. It was edited from Suva and Lae and printed at Unitech and later in Suva. It was widely distributed, and free.

This year, it has returned home to the Mathematics Education Centre in Lae. There will be a new format, exciting articles, more of emphasis on research and high-quality writing and an attempt to keep teachers informed of developments in Mathematics Education. There will be a section for reports of activities of Mathematics Clubs, groups, and Associations - and this is where you, as Mathematics teachers/educators/ curriculum developers in Papua New Guinea come in. The journal is potentially very useful as a medium for reporting research, exchanging teaching strategies, discussing curriculum revision, and helping teachers feel as if they were part of something instead of on their own.

Prime is being consiously kept Regional in tone, and hopefully, Papua New Guinea will not completely over power it.

THE SOUTH PACIFIC MATHEMATICS AND LOGIC COMPETITION

In 1975, Colin Meek of the Mathematics Education Centre introduced a Mathematics and Logic Competition in Papua New Guinea, the main aim of which was to increase students interest in mathematics and logic and to develop their problem solving abilities. At the same time, Peter Sanders was involved in the Eastern Africa Regional Contest. When Peter moved to the University of the South Pacific in early 1977, the South Pacific Mathematics and Logic Competition truely went international. It evolved to what we have now.

As originally designed it was 30 multiple choice questions with 75 minutes to complete it and a limit on the number of entrants from each school. The organizers, however, soon realized that this type of paper did more to encourage inspired guess work than inspired problem-solving. Certainly, the paper did not seem to be an effective way of measuring problem-solving abilities. This led to the evolution of the two-paper system presently in use. Paper 1 is a qualifying paper-short multiple-choice questions, marked by the local school, so that as many as wanted to could enter. The school then selects the top 15 - or those who score over a certain mark (to a limit of 15 per school), to continue on to Paper 2. Paper 2 is a problems-type paper where working, as well as the answer, must be shown. Paper 2 is marked in Suva, Honiara, or Lae. There is an upper age limit. Grade 12 is normally excluded, not for any esoteric reason, but because it was generally felt that they were tested and examined enough in their last year, that one more would be a burden. Unofficially, there have been Grade 12 entrants who wished to try their luck.

Questions are mostly plagarized from other sources, with rewriting done to add "local colour". The levels of difficulty vary, but an attempt is made to ensure that it is the logic and problem-solving aspects that are tested, and not so much in mathematics-recall. (see examples in figure 2.)

Judging from the letters received over the past two years, interest is high and criticism flows freely in Papua New Guinea. It looks as if it is time for the Mathematics Panels to take a hand in the developing and running of such a competition. This is a crucial year to the organizers - Peter Sanders finishes at University of the South Pacific in December, Colin Meek is undecided in Honiara, and I will be away from September through December, 1982. It is the perfect time for those interested in Mathematics in Papua New Guinea to get involved. Copies of the 1980 and 1981 papers are available on request.

HELEN WHIPPY
MAY, 1982.

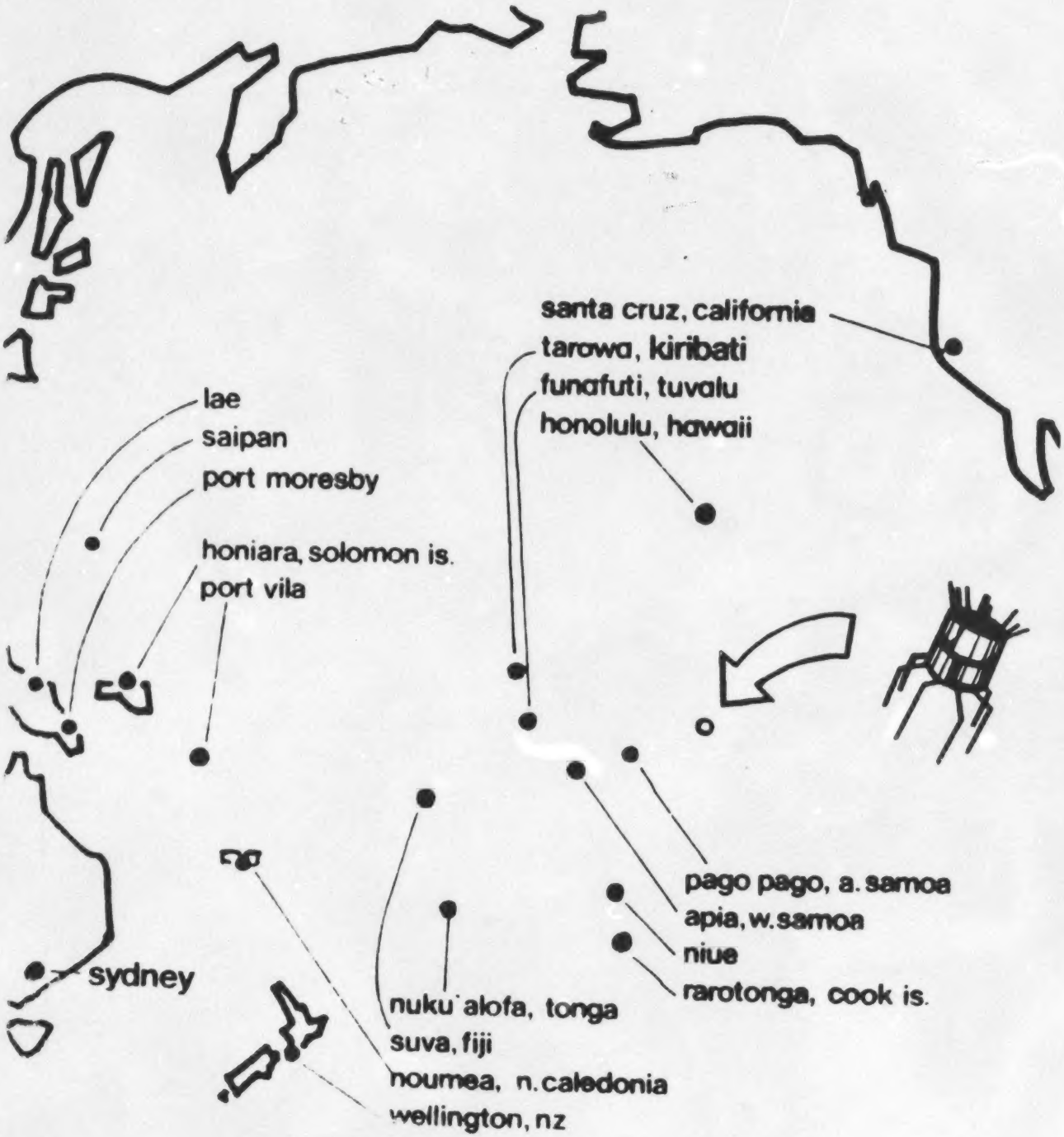


FIGURE 1: PEACESAT Terminals in operation.

Question One

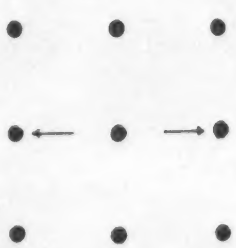
Diagram A

(i) In diagram A, move the two dots marked with an arrow to make 4 lines each of which contain 4 dots.

(5 marks)

Diagram B

(ii) In diagram B, move the two dots marked with an arrow to make 10 lines each of which contain 3 dots.



(6 marks)

Question Thirteen

The Beesbush flower has a stem that climbs up and around a tree in a clockwise direction. The Tangweed flower climbs up and around the same tree but in an anti-clockwise direction. They both start growing at the same point on the ground.
Before they reach the first branch the Beesbush has made 5 complete twists and the Tangweed has made 3 complete twists.
How many times do they cross each other (not counting the top or the bottom)?

(a) 2 (b) 4 (c) 6 (d) 7 (e) None of these.

FIGURE 2: Examples of problems from previous SPMLC papers.

PAPUA NEW GUINEA

UNIVERSITY OF TECHNOLOGY

MATHEMATICS EDUCATION CENTRE

CONFERENCE 5-6 MAY 1982.

TITLE: Development and Implementation of (Mathematics) Textbooks in PNG.

A case study of the development of a set of Provincial High School Mathematics Textbooks.

ABSTRACT: This study is intended to examine the development phases in the writing of a set of Mathematics textbooks for Grades 7 and 8 in PNG Provincial High Schools over a period of five years (1976-80) followed by close study of four years (1981-84) implementation phase.

It is hoped that the problems of writing, trialling and revision will be identified through studies of questionnaire responses, syllabus advisory committee discussions, short visits to some schools, interviewing some Mathematics teachers and testing some units.

The implementation process will be examined in terms of in-service strategies employed and mechanisms established to ensure that maximum classroom use was made of the textbooks.

It is hoped that data received will provide information to the Department of Education to provide some measure of the influence of these textbooks on teachers' teaching and pupils' learning of Mathematics. From this study common threads between the development of Mathematics textbooks and textbooks in general will be identified in the hope that those following a similar process of textbook development in other subjects, and indeed in other education systems, may benefit from what will be learned from this textbook project.

NOTE: This study at the moment is at the proposal stage. The Research Unit of the Department of Education has shown interest and will provide the necessary finance. The aim of presenting paper at this stage is to gauge essential directions from experts to make this proposed study reliable and meaningful to the Department of Education.

GODFREY YERUA
Assoc. Senior Curriculum Officer - Maths.
Curriculum Unit
April 1982.

PAPUA NEW GUINEA
UNIVERSITY OF TECHNOLOGY
MATHEMATICS EDUCATION CENTRE
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development of Mathematics textbooks and textbooks in general will be identified in the hope that those following a similar process of textbook development in other subjects, and indeed in other education systems, may benefit from what will be learned from this textbook project.

INTRODUCTION

At the present stage of Papua New Guinea development many people are concerned about the standard of education their children receive. Many of them believe that the standard is falling but do so without any statistical evidence. Many of us (curriculum developers and teachers in the field) feel that appropriate textbooks for teachers and pupils will promote pupils' academic achievement. Consequently, many teachers are demanding that appropriate textbooks be provided to accompany the present syllabuses. Educationalists and those who came through the present education system feel that textbooks for both teachers and pupils will have a significant effect on pupils' academic achievement. It is appreciated however, that textbooks are only part of the solution to the present concern over low standards in education.

Due to the constant requests by teachers for textbooks, the Department of Education is now arranging to produce textbooks for most subjects in schools. This will happen when manpower and finance are available during the next 10-15 years. Textbooks are easy to use but they are costly and time consuming to produce. Mathematics textbooks for high schools are already being produced for Grades 7-10.

One should bear in mind that not all textbooks produced for a particular grade level are usable in PNG. Often there is a lack of availability of materials prescribed in the texts and/or because the English used in the texts is too hard to understand. There are other numerous reasons. Therefore, there is a need to evaluate the use of the Mathematics textbooks which are being produced by the Department of Education. The information gathered from such an evaluation will be

extremely useful in guiding the Department of Education in future textbooks production.

On this premise I am proposing to conduct such evaluation research on the Grades 7 and 8 Mathematics textbooks. Experts from developed countries tell us that textbooks do assist pupil learning. This research would be aimed at establishing whether this is true for Papua New Guinea.

OUTLINE

The study would consider teachers attitudes towards Mathematics textbooks and the levels of pupils achievement in classes where textbooks are used. To effectively study this situation, I will administer 4 or 5 different paper and pencil tests to pupils during short visits to some schools; set one test at the end of the year covering all the topics in each grade; interview maths teachers, subject masters and headmasters in a representative sample of schools. A sample of ten schools is considered to be sufficient for the study. Questionnaires will be sent to all other high schools for maths teachers, subject masters and headmasters to answer.

There will be 3 questionnaires, one for each group; maths teachers, subject masters and headmasters. These questionnaires will collect data on the availability of materials prescribed in the textbooks; teachers' background, experience, qualifications, in-service experience and views on Mathematics textbooks.

Ten schools have been selected for personal visits. I will administer written tests and interview teachers. These 10 schools have been selected with regards to the following factors: urban/rural location, agency (church/government), boarding/day and co-educational/non co-educational. Pupils will be selected randomly within a class to do one of the (4 or 5) tests. A random sample of about 300 pupils per test should be sufficient for this study.

The schools selected are as follows:

Fatima High School (WHP)
Asaroka High School (EHP)
Tari High School (SHP)
Vunapope High School (ENBP)
Hutjena High School (NSP)
Bugandi High School (Morobe P)
Cameron High School (MBP)- SSCEP
Gordon High School (NCD)
Hagita High School (MBP)
Kerema High School (Gulf P)

These schools are also trialling Grade 9 materials which is considered an added reason for visiting them.

The results for this study is expected to show us where the weaknesses are in using the textbooks. Once this is known, recommendations will be made to the Department of Education to overcome such weaknesses in the design and implementation of textbooks.

HISTORY BEHIND THE MOW BOOK SERIES

PRE-INDEPENDENCE

Mathematics learning in PNG high schools during the period of pre-independence was based on Australian syllabus. During that period two syllabuses were used at different times. In 1975, the third syllabus was prepared and distributed to schools for trialling in a draft form. There was no prescribed text for teachers to use with the exception of a few overseas texts on which they could lay their hands.

POST-INDEPENDENCE

Curriculum development activities were at an early stage in 1975 when the present Curriculum Unit was established. That year, the Department of Education established a Mathematics Syllabus Committee to advise the Secretary for Education.

During the same year the Department of Education decided to recruit a Curriculum Specialist through UNESCO to write new syllabusses. In the same year, the UNESCO Curriculum specialist, Mr. Murray Britt arrived and was based at Goroka Teachers' College. The 1975 draft syllabus was discussed at a meeting of the Syllabus Advisory Committee in November 1975. Consequently, a new syllabus for Grades 7 and 8 was developed.

Mr. Murray Britt's role in the development of this new syllabus was to write the syllabus objectives in behavioural terms and to include teaching suggestions for each objective.

The new syllabus was written and distributed to all high schools by the beginning of 1976.

In May 1976, a syllabus/teachers' guide for grades 9 and 10 was also prepared by Mr. Britt following a Syllabus Advisory Committee meeting. This new syllabus was written in the same format as that of the Grades 7 and 8 syllabus. In the same year, it was issued to all schools for a two year trial period.

In July 1976, short workshops were conducted throughout the country in an effort to properly establish new programmes. At these workshops, many teachers in all areas expressed their urgent need to have text materials based on the new syllabuses which should be relevant and appropriate to the PNG situation. At the same time, a survey, (Roberts 1976) conducted by Dr. N. Roberts of the UPNG (now at PNGUT), revealed that the majority of teachers in the country felt that the supply of text material was priority need.

PUPILS' WORKBOOKS

As a result, a proposal was made to produce text material in the form of sets of pupils' booklets. These pupils' booklets were developed for Grades 7 and 8.

During late 1976 and in 1977 Grade 7 material was prepared and trialled in four highlands schools. Restriction of the trial to these schools was due to economical and manpower limitations.

By January 1978, all Grade 7 pupils in the country had access to the first set of books for Grade 7. The completed set comprised of fifteen pupils' booklets. The first Grade 8 pupils booklets reached schools in February 1978, the last booklets (13 altogether) didn't reach schools until March 1980. The curriculum development activities were slowed down because of wider trialling and more detailed discussion among groups of teachers, lecturers and others who had formed Mathematics Education Groups in four centres (Port Moresby, Lae, Goroka and Rabaul) during 1978.

These Mathematics Education Groups were formed for the following reasons:-

(a) participation in the preparation of curriculum materials for both provincial high schools and community schools.

(b) to monitor the trialling of pupils' booklets.

(c) to provide feedback and suggestions on trial materials.

(d) participation in the preparation of in-service activities with in their provinces.

Since then new Mathematics Education Groups were formed and now we have about 8 groups in total.

TEXTBOOKS:

The Department of Education was concerned about the longevity of these booklets in schools because of storage and the logistics of using them. Thus, it was decided to further develop the Grades 7 and 8 booklets as a two-volume text respectively. The text has been called "Mathematics Our Way".

The first volume of the Grade 7 text (MOW) - a high school series - has been produced and sent out to schools (1981). The second volume of the Grade 7 text was sent to schools early this year. The first volume of the Grade 8 text is being sent out to schools now. The second volume of the Grade 8 text will be sent to schools later this year as soon as it arrives from Singapore pinters.

First draft of texts for Grades 9 and 10 will also be written by Mr. Murray Britt working on contract from Auckland. Materials for Grade 9 text are being trialled through the Maths Education Groups at the moment and for Grade 10 it will be trialled next year. Texts for Grades 9 and 10 are expected to be in schools in 1984 and 1985 respectively, if everything run as smoothly as planned.

Plans for the production of Mathematics texts for community schools Grades 4, 5 and 6 to link with MOW 7 are also well underway. The writer, Mr Bryn Roberts is here on 3 years contract to write texts for Grades 4, 5 and 6. We hope to get these textbooks into community schools in 1984 or 1985. This project is part of Education II Project.

GODFREY YERUA
Curriculum Unit
Department of Education

References:

Roberts, A.M. 1977: "Mathematics in P.H.S. 1976," PNGJE, Vol 13, No.1 pp.7-15

MATHEMATICS EDUCATION CONFERENCE

On the 5-6 May a conference on mathematics education will be hosted by the Mathematics Education Centre. The venue for the conference will be the Audio-Visual Theatre. The program is given below. All are welcome to attend.

PROGRAM

Wednesday 5th May

8:15 - 8:30	Opening of Conference.
8:30 - 9:15	Mr. Glen Lean (Maths Dept.) "The Rotation and Development of Surfaces - A Study of a Piagetian Spatial Task with PNG Secondary School Pupils".
9:15 - 9:45	Dr. Bob Hyde (Maths Dept.) "A Hierarchical Scheme for the Mathematical Generalization Process".
9:45 - 10:00	Morning Tea.
Chairperson -	Rod Selden
10:00 - 10:30	Dr. Lilia Dube (Maths Dept.) "An Extraneous Role for the VTR in Educational Research".
10:00 - 11:00	Dr. Gilah Leder (Monash University) "Attitudes to Success and Failure in Mathematics".
11:00 - 11:30	Mr. Philip Clarkson (Mathematics Education Centre) "Factors Affecting Mathematics Learning in National High School".
11:30 - 1:00	Lunch.
Chairperson -	Peter Sullivan
1:00 - 1:30	Mr. Bob Roberts (Education Dept.) "Community School Mathematics: The International Connection".
1:30 - 2:00	Br. Lawrence Ros (Goroka Teachers College) "The Use of Algorithms in Year 7".
2:00 - 2:30	Mr. Godfrey Yerua (Education Dept.) "Development and Implementation of Math Textbooks".
2:30 - 3:00	Afternoon Tea.
3:00 - 4:00	Dr. David Shields (Maths Dept.) "Using CAI at Unitech". (meet in the ABS Computer Room)

Thursday 6th May

8:15 - 8:45	Mr. Bryn Roberts (Education Dept.)
8:45 - 9:15	Mr. Rod Selden (Maths Dept.) "Time Concepts in Papua New Guinean Traditional Cultures".
9:15 - 9:45	Mr. Peter Sullivan (Mathematics Education Centre) "The Development of Time Concepts and the Use of Comparatives Among Papua New Guineans".
9:45 - 10:00	Morning Tea.
Chairperson -	Helen Whippy
10:00 - 10:30	Mr. Peter Clarkson "The Use of Academic Subjects in a Service Capacity".
10:30 - 11:00	Mr. John Gough (University of Papua New Guinea) "Arguments for Learner - Centred, Process-Oriented Maths Teaching".
11:00 - 11.30	Mr. Harold Fredericks (Goroka Teachers College) "A Review of the Problem Solving Literature".
11:30 - 1:00	Lunch.
1:00 - 1:30	Mrs. Helen Whippy (Maths Dept.) "Regional Cooperation in Maths Education". (meet in the Committee Room of Library).
1:30 - 2:00	Close.

PHIL CLARKSON,
Director,
Mathematics Education Centre.

The following M.E.C. Reports have been published:

No. 1: A. EDWARD 'Report for 1978, including a Short History of the Mathematics Learning Project and Mathematics Education Centre up to 1977', 1978.

No. 2: A.J. BISHOP 'Spatial Abilities in a Papua New Guinea Context', 1978.

No. 3: C. MEEK and N. FERIL 'A Pilot Study into the Level of Comprehension of Some Words taken from the Papua New Guinea Community Schools Mathematics Syllabus', 1978.

No. 4: A.J. BISHOP 'Developing Spatial Abilities', 1977.

No. 5: A. EDWARDS 'A Study of the Issues Involved in Establishing the Mathematics Education Centre and a Report on its Work from December 1978 to May 1979', 1979.

No. 6: A. EDWARDS and A. BAJPAI 'Preliminary Report on the Basic Arithmetic Test Carried out in Post-Secondary Institutions in Papua New Guinea in 1979', 1979 (out of print).

No. 7: A. EDWARDS 'Final Report on the Establishment of the Mathematics Education Centre', 1979 (out of print).

No. 8: G.A. LEAN 'The Training of Spatial Abilities: A Bibliograph', 1980.

No. 9: G.A. LEAN 'An Investigation of Spatial Ability Among Papua New Guinea Students', 1980.

No. 10: G.A. LEAN and M.A. CLEMENTS 'Spatial Ability, Visual Imagery and Mathematical Performance', 1980.

No. 11: P.L. JONES 'The Introductory Mathematics Programme at the Papua New Guinea University of Technology - A Survey of Student Opinion', 1980.

No. 12: P.L. JONES 'A Study of Proportional Reasoning', 1980.

No. 13: M.A. CLEMENTS and G.A. LEAN 'Influences on Mathematical Learning in Papua New Guinea: Some Cross - Cultural Perspectives' 1981, (2nd edition).

No. 14: M.A. CLEMENTS 'Careless Errors Made by Sixth-Grade Children on Written Mathematical Tasks', 1981 (2nd edition).

No. 15: P. SULLIVAN 'The Standards Debate: An Alternate Strategy', 1981.

No. 16: P.L. JONES 'Mathematical Language and the Learning of Mathematics in Papua New Guinea', 1981.

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